

THE INTERACTIONS BETWEEN π -MESONS AND NUCLEONS^{1,2,3}

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INTRODUCTION

During the some seven years which have elapsed since the first observation of artificially produced π -mesons at Berkeley, we have seen a very rapid development of elementary particle physics. There are now at least a dozen accelerators capable of producing π -mesons, including several which have created or are expected to create heavier particles. As a result of the work done with these machines, it is perhaps fair to say that we have at least a qualitative knowledge of the elementary interactions of mesons with nucleons at low energies.

On the other hand, our understanding of the π -meson interactions has in no sense kept pace with our knowledge of these phenomena. The lack of marked success of the purely field theoretic approaches to pion-nucleon processes should probably have been anticipated. Indeed, the very existence of the multitude of heavier unstable particles casts doubt on the hope for success of present day field theories.

In spite of these difficulties of a more fundamental approach, it has been found possible to simplify greatly the experimental data by applying to pion reactions various levels of phenomenological discussion. In many cases this has involved no more than the use of a few general quantum mechanical principles combined with some physical concepts borrowed from the more familiar field of nuclear physics. In other cases, more elaborate models have been suggested, and some of these have proven quite useful.

It will be our primary purpose in the present article to attempt to summarize these theoretical concepts, which have been employed in the study of meson properties. Our model is based on three assumptions:

- (a) The pion-nucleon interactions have a finite range.
- (b) Charge independence is valid for these phenomena (i.e., I -spin is conserved).

¹ The survey of literature pertaining to this review was completed in June, 1954.

² C.I.T. as used in this article refers to California Institute of Technology; Carnegie Institute of Technology is referred to as Carnegie Tech.

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- (c) The state ($I=3/2$, $j=3/2$, $l=1$) of the pion-nucleon system is one of especially strong ("attractive") interaction.

The good agreement of this model with present experiments for "low energies" is noted, and the relations of more detailed and basic theories to this model and to the data are discussed.

We shall begin on an almost purely phenomenological level, combining a few general principles of quantum mechanics with some simple physical ideas (see the next section). The presently available experimental data are discussed in terms of their relation to these principles. Finally, the successes of the more sophisticated approaches will be studied.

SOME PHYSICAL CONCEPTS APPLYING TO PIONS

General principles.—We shall be primarily concerned with two-body collisions involving the production (and absorption) and scattering of π -mesons from nucleons. The most studied of these processes fall into three classes:

[Scattering:—(S)]

$$[6] \quad \pi + N \rightarrow \pi + N.$$

(We use π and N to denote pions and nucleons, respectively, irrespectively of their charge states.)

[Photoproduction:—(P_γ)]

$$[4] \quad \gamma + N(\leftrightarrow) \pi + N$$

[Production in Nucleon-Nucleon Collisions:—(P_n)]

$$[7] \quad N + N(\leftrightarrow) \pi + N + N.$$

At sufficiently high energies we may expect additional pions to be produced in each of these processes. Also, other unstable particles may appear (1). The number in brackets at the left of these reaction equations indicates the number of reactions which are obtained by enumerating the possible pion and nucleon charge states.

We shall be most concerned with these reactions at low energies. By "low energies" we specifically mean energies such that the de Broglie wavelength, λ , of the meson (in the barycentric system) is not small compared to the range of the interaction involved. We do not expect this range to be much greater (except for Coulomb interactions) than the Compton wavelength

$$\hbar/\mu c,$$

where μ is the rest-mass of the meson. As a result of this, we may suppose that orbital angular momenta greater than $l\hbar$, where

$$l = (1/\lambda) \left(\frac{\hbar}{\mu c} \right) \quad 1.$$

will not play an important role in the reactions considered.

To illustrate this, in Figure 1 we plot l as determined from Equations 1

versus the meson energy in the laboratory frame of reference for the scattering process (S) above. It is evident that for many processes at most one

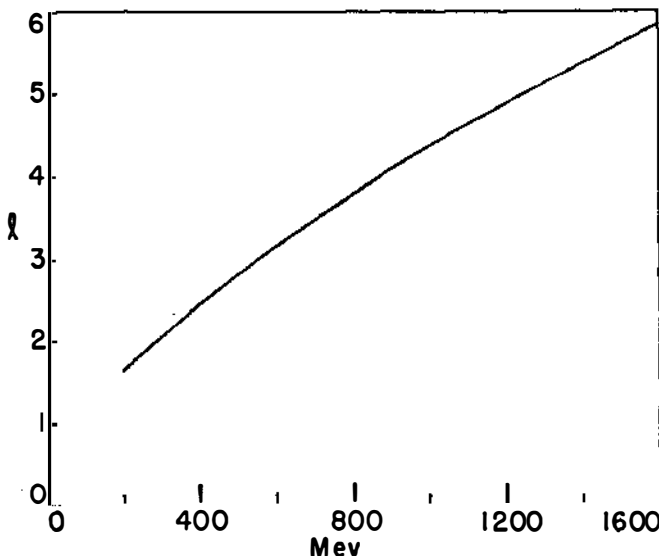


FIG. 1. The highest value of the angular momentum in units of \hbar which is expected for pion-nucleon scattering, versus the pion energy in the laboratory system.

or two values of l need be considered for fair ranges of energy if our supposition concerning the range of the interaction is correct. From the subsequent discussion it will appear that this conclusion seems to be quite correct, with a single state of unit orbital angular momentum (P -state) playing a very important part.

Just as in nuclear physics, the conclusion that the range of interaction is of limited extent has specific consequences. For instance, when only one state of orbital angular momentum is important for an emitted (or absorbed) particle, the energy dependence of the cross section is uniquely determined at "low energies."⁴ The energy dependence is summarized for the processes of interest to us in Table I. Here q is the momentum of the meson in the barycentric system. When at most one or two states of angular momentum are important we are led to expect a simple dependence of the cross sections on

⁴ To see this, let $T(r)$ be the scattering matrix for the emission of a particle whose space coordinate is r . Also let $T(r)$ vanish for $r > R$ and suppose that only the angular momentum l is of importance. Then the amplitude for emitting the particle into a plane wave state $\phi_q(r)$, where q is the momentum, is

$$A = \int \phi_q^*(r) T(r) d^3r.$$

Since the l th partial wave of ϕ_q varies as $(qr/\hbar)^l$ for $r < \hbar/q$, we have $A \sim q^l$ for $\hbar/q > R$.

TABLE I

THE DEPENDENCE OF THE CROSS SECTION AT "LOW ENERGY" ON THE MOMENTUM OF THE MESON WHICH IS ABSORBED, EMITTED, OR SCATTERED*†

Type of cross section	Absorption	Emission into two particle state	Emission into three particle state	Scattering
Dependence of cross section on momentum	q^{2l-1}	q^{2l+1}	q^{2l+4}	q^{4l}
Dependence of "matrix element" T , on momentum	q^l	q^l	q^l	q^{2l}

* q is the momentum of the meson, l is its angular momentum.

† When some of the emitted particles interact strongly, the q -dependence may be modified, but often in a simple manner. (See section on π -MESON PRODUCTION IN NUCLEON-NUCLEON COLLISIONS.)

energy. (For more than one l -state, we expect linear combinations of the corresponding terms in Table I.) In Figure 2 we present a comparison of several experimental total cross sections with some simple power law curves. Except for the process ($P_\gamma: \gamma + p \rightarrow \pi^+ + n$) a P -state power law appears reasonably satisfactory. In the latter case a linear combination of S - and P -waves is required even for rather low γ -ray energies. We shall develop these considerations in more detail in subsequent sections.

A second consequence of our conclusion that few orbital angular momentum states are expected to be important at low energies is that angular momentum and parity conservation will be significant for our considerations. This point will be developed in more detail as we discuss the reactions individually.

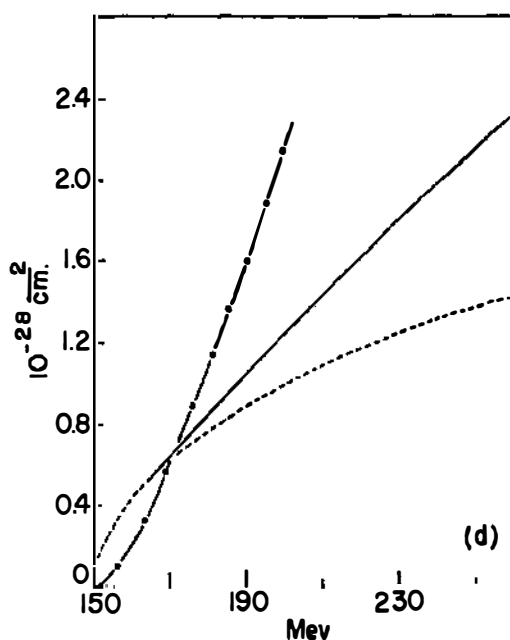
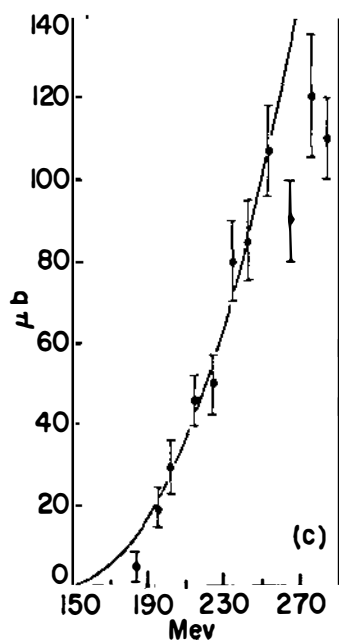
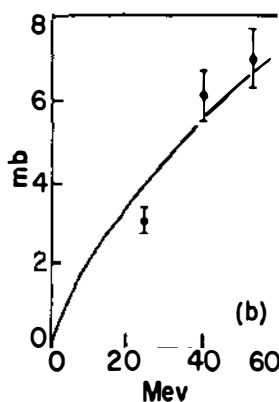
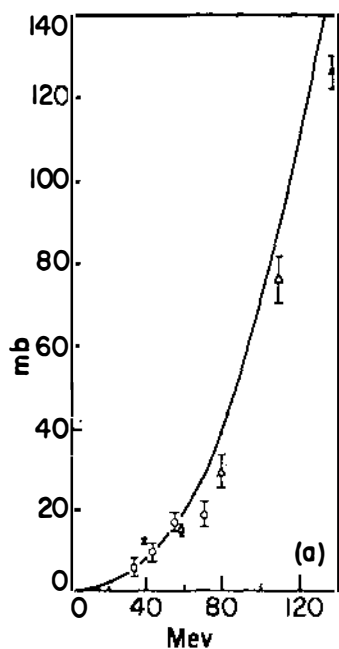
There is, however, one particularly important aspect of angular momentum and parity conservation which we now describe. The π -meson is pseudoscalar (2, 3) which implies (by definition) that the emission of a single meson by a single nucleon must be into a state of odd orbital angular mo-

FIG. 2. [a] π^+ -proton scattering compared to $\sigma \sim q^4$, where q is the meson momentum in the barycentric system. The points are from: Φ —Leonard, S., and Stork, D., *Phys. Rev.* **93**, 568 (1954). The remaining are from Table III: [$\frac{1}{2}$ (d), $\frac{1}{2}$ (e); $\frac{1}{2}$ (g)].

[b] $\pi^+ + d \rightarrow p + p$. The Durbin, R., Loar, H., and Steinberger, J., [*Phys. Rev.* **84**, 581 (1951)] measurements are compared with $\sigma \sim q$.

[c] $\sigma(\gamma + p \rightarrow \pi^0 + p)$ as measured by Goldschmidt-Clermont, Y., Osborne, L., and Scott, M., *Phys. Rev.* **89**, 329 (1953) is compared with $\sigma \sim q^3$.

[d] —experimental $\sigma[\gamma + p \rightarrow \pi^+ + n]$ compared with $\sigma \sim q$ (— — — —) and $\sigma \sim q^3$ (— · — · —). See Section on PHOTOPRODUCTION OF π -MESONS FROM NUCLEONS for details.



mentum. The total angular momentum must remain $j=1/2$, however, since this is the value for the nucleon initially. The only odd orbital state of the meson-nucleon system with $j=1/2$ is the $l=1$, or P -state. Thus the simple emission (or absorption) process must always lead to (or from) a P -state.

This naturally does not represent a selection rule for physical processes, since simple emission or absorption cannot occur alone. On the other hand, if these simple emission and absorption processes represent the important steps in developing a physical emission or absorption, then it is quite reasonable that the P -state should be predominant (at low energies). This is certainly suggested by the experiments (see Fig. 2 and the subsequent sections) and is not at all incompatible with many expectations from field theory.

We may now summarize the arguments of the present section. Admitting the possibility of exceptional cases, we suppose meson reactions to take place predominantly in P -states at "low energies." Smaller admixtures of other orbital states are of course expected. The dependence of the cross section on energy is uniquely determined at sufficiently low energies by the relevant states of orbital angular momentum. Finally, we must consider selection rules and other consequences of angular momentum and parity conservation. The purpose of most phenomenological analyses of meson properties has been to determine to what extent this simplified model is (or is not) adequate.

The hypothesis of charge independence.—The hypothesis of charge independence has provided a useful simplification for the study of pion phenomena. This principle seems to have had its origin in a suggestion of Breit & Feenberg (4), who proposed in 1936 that the n - n , p - p and n - p nuclear forces might be the same for states of equal angular momentum and parity (which we now know to be at least approximately correct at low energies). This suggestion was based on the similar binding energies and scattering properties of neutrons and protons. To be more specific, we note that a state of the two-body system may be labeled by the quantum numbers⁵ (j , S , π , Q), where Q is the charge, S the spin, j the total angular momentum and π the parity. The charge independence hypothesis asserts that the interaction in this state is independent of Q .

Kemmer (5) showed how to construct a meson theory which would always lead to charge independent nuclear forces. Heitler (6) later showed that the Kemmer theory should lead to selection and intensity rules for meson reactions. In view of the uncertainty of meson theories it is desirable to divorce the charge independence hypothesis from meson theory. This may be readily done in a manner which gives a simple interpretation to charge independence (7). In its broadest form (proposed to date) we may say that charge independence implies that neutrons and protons are completely equivalent physically, to the extent that the "weak interactions" (for example, elec-

⁵ The spin S must be conserved if we accept charge independence.

tromagnetic and β -decay interactions) are of negligible importance. This means that a wave function which is constructed as a linear combination of neutron and proton wave functions must be physically equivalent to the wave function of either a neutron or proton.

Expressed in mathematical form, we may say that a general unitary transformation which replaces the wave function of each neutron or proton of a system by a linear combination of neutron and proton wave functions must leave the physical properties of the system unchanged. Except for a phase factor, this transformation is equivalent to (isomorphic to) a spin representation of the rotation group in three dimensions (this space has been called "charge space"). At this point we may draw a useful analogy. The invariance of a physical system with respect to rotations in ordinary space implies the conservation of angular momentum, whose operators are the generators of the rotation and whose eigenvalues are called the "isotopic spin." It is evident from the invariance with respect to "charge rotations," as just discussed, that the generators of these rotations will also be conserved. From the analogy to angular momentum, it is clear that these "charge rotation operators" will be formally identical to the angular momentum operators, so we may use the mathematical apparatus for the latter without modification.

In particular, we may introduce a two-component wave function for the "nucleon," its components referring to neutron and proton states. The "charge rotations" are induced by three two-dimensional matrices, τ_1 , τ_2 , and τ_3 , which are formally equivalent to the Pauli-spin matrices. The τ 's are components of a vector τ (with respect to charge rotations) in the three dimensional charge space. For a system of several nucleons a total isotopic spin I may be constructed just as can a total spin S for their ordinary spin. To use a definite representation, we shall suppose that a proton has isotopic spin "up" and a neutron isotopic spin "down." (This assignment is of course arbitrary and is often inverted.)

The principle extends itself uniquely to unstable particles which may be emitted or absorbed singly by nucleons. For instance, consider the emission of a π^+ -meson by a proton. Charge independence states that this emission process is unchanged when by the "rotation" the proton wave function is replaced by a linear combination of proton and neutron wave functions. It is evident that this "rotation" must replace the π^+ eigenfunction by a linear combination of π^+ , π^- , and π^0 wave functions, since a neutron cannot emit a π^+ -meson. This is a three dimensional [irreducible (7)] representation of the rotation group in three dimensions, so the meson has an "isotopic spin" of unity with an isotopic angular momentum operator τ . To summarize, each meson has an isotopic spin of unity, each nucleon an isotopic spin of one-half. A system containing several nucleons and mesons may be resolved into states of total isotopic spin I , in exact analogy with the corresponding problem for ordinary angular momentum. The state I is $(2I+1)$ -fold degenerate, the substates being dynamically equivalent.

If the third component of τ is diagonal, the meson wave functions transform under rotations as the spherical harmonics of order unity in charge space:

$$\begin{aligned}\phi(\pi^+) &\rightarrow Y_1^1 \\ \phi(\pi^0) &\rightarrow Y_1^0 \\ \phi(\pi^-) &\rightarrow Y_1^{-1}.\end{aligned}\tag{2}$$

The meson and nucleonic charge operators are, respectively,

$$Q_\pi = \tau_3, \quad Q_N = \frac{1}{2}[1 + \tau_3],\tag{3}$$

in units of the charge on the proton.

The total I -spins ("isotopic spins") of some simple meson-nucleon systems are listed in Table II. Each of these I -states represents a constant of

TABLE II
POSSIBLE VALUES OF THE I -SPIN FOR SOME MESON-NUCLEON SYSTEMS

System	Single nucleon	Single pion	Two nucleon	Pion—nucleon	Pion—Two nucleon	Two pions	Two pions—One nucleon
I -spin values	1/2	1	0, 1	1/2, 3/2	0, 1, 1, 2	0, 1, 2	1/2, 1/2, 3/2, 3/2, 5/2

the motion for the dynamical behavior of the corresponding system. The $(2I+1)$ substates for each of these are equivalent, which means that the total number of different reactions is greatly reduced. For instance, in the first part of this section (see *General principles*) it was stated that there were six scattering cross sections (S). Reference to Table II indicates only two I -states; therefore only two, rather than six, reactions need be found. For processes P_π Table II indicates that $I=0, 1, 1$ and there are consequently only three, rather than seven, independent processes.

We must finally observe that charge independence in its present form is not rigorous because of the "weak interactions" which violate it (electromagnetic interactions imply a preferred direction in charge space, which keeps us from losing track of the distinction between neutrons and protons, etc.). This means that transitions between different I -states will occur; however, these are presumably of only secondary importance for the phenomena which we are considering (with the exception of the specifically electromagnetic interactions such as photoproduction of pions).

The hypothetical state of strong interaction for the meson-nucleon system.—It is convenient to label states of the one meson-one nucleon system by the isotopic spin I , the angular momentum j , and the orbital angular momentum l , as (I, j, l) . There is a suggestion from meson theory that the $(3/2, 3/2, 1)$ -state should be one of particularly strong interaction at certain energies.

Brueckner (8) has put this into the form of an explicit hypothesis by assuming that the scattering in this state passes through a resonance at a meson energy of about 200 Mev in the laboratory system. This hypothesis leads to a variety of implications for pion phenomena which will subsequently be discussed in more detail. There is an increasing amount of evidence that the resonance does indeed exist.

THE SCATTERING OF PIONS BY NUCLEONS

The elementary scattering processes which have been studied are:

$$\pi^+ + p \rightarrow \pi^+ + p \quad (S_+)$$

$$\pi^- + p \rightarrow \pi^- + p \quad (S_-)$$

$$\pi^- + p \rightarrow \pi^0 + n. \quad (S_0)$$

The differential cross sections for these processes we shall designate by σ^+ , σ^- and σ^0 , respectively, whereas we shall use σ_T^+ , σ_T^- , σ_T^0 for the total cross section. (We shall consistently represent differential and total cross sections by σ and σ_T , respectively.)

We have indicated in Figure 2 that P -wave scattering plays a dominant role in the pion scattering over a considerable energy range. This does not in any way imply that other partial waves are negligible, a point to which we shall return.

A great deal of effort has been put into the experimental study of the pion-proton scattering. The presently available total cross sections are summarized in Figure 3. In Table III we give the experiments and references from which these points are obtained.⁶ Of particular interest are the cross sections in the vicinity of 200 Mev recently obtained at Carnegie Institute of Technology [Table III, references (f) and (g)] and those above 300 Mev measured at Brookhaven [Table III, references (h), (j) and (k)]. The cross sections plotted are σ^+ and $\sigma_T^{(-)} \equiv \sigma_T^- + \sigma_T^0$.

It should be mentioned that the cross sections of Figure 3 include inelastic scattering (*i.e.*, with the production of one or more additional mesons). This becomes energetically possible for pion energies above 200 Mev in the laboratory system. In the vicinity of one Bev the cross sections are probably largely inelastic (9).

It is instructive to resolve the cross sections of Figure 3 into cross sections $\sigma_{1/2}$ and $\sigma_{3/2}$ for the pure isotopic spin substates $I=1/2$ and $3/2$, respectively. This is easily done using the relations

$$\begin{aligned} \sigma_T^+ &= \sigma_{3/2} \\ \sigma_T^{(-)} &= \frac{1}{3}[\sigma_{3/2} + 2\sigma_{1/2}], \end{aligned} \quad 5.$$

or

⁶ We are deeply indebted to those workers who have supplied us with detailed information of their unpublished experiments. In particular we should like to thank Drs. J. Ashkin., O. Piccioni, and their collaborators.

TABLE III
REFERENCES TO THE EXPERIMENTAL POINTS OF FIGURE 3*

Pion energy in lab system in mev	Refer- ence	Experiment (measured cross sections are indicated)
40	(a)	Perry, J. P., and Angell, C. E., <i>Phys. Rev.</i> , 91 , 1289 (1953) [σ^+ ; counter telescope in coincidence with beam defining telescope]
40	(b)	Barnes, S., Angell, C., Perry, J., Miller, D., Ring, J., and Nelson, D., <i>Phys. Rev.</i> , 92 , 1327 (1953) [σ^- ; counter telescope]
34	(c)	Roberts, A., and Tinlot, J., <i>Phys. Rev.</i> 90 , 951 (1953) [σ^e , γ -rays counted singly and in coincidence]
58, 65	(d)	Bodansky, D., Sachs, A., and Steinberger, J., <i>Phys. Rev.</i> , 93 , 1367 (1954) [σ^+ , σ^- , σ^e ; scintillation counters, liquid hydrogen target]
78, 110, 120	(e)	Anderson, H., Fermi, E., Martin, R., and Nagle, D., <i>Phys. Rev.</i> , 91 , 155 (1953) [σ^+ , σ^- , σ^e ; scintillation counters, liquid hydrogen target]
135 to 250	(f)	Ashkin, J., Blaser, J., Feiner, F., Gorman, J., and Stern, M., <i>Phys. Rev.</i> , 93 , 1129 (1954). [$\sigma_T^{(-)}$ transmission, using liquid hydrogen]
135 to 196	(g)	Ashkin, J., Blaser, J., Feiner, F., Gorman, J., and Stern, M., <i>Phys. Rev.</i> , 93 , 1129 (1954) [σ_T^+ ; as above]
28, 340, 450	(h)	Lindenbaum, S. J., and Yuan, L. C., <i>Phys. Rev.</i> (In press), also <i>Proceedings of 1954 Rochester Conference</i> [σ_T^+ ; transmission, C-CH ₂ subtraction]
340, 450	(j)	Lindenbaum, S. J., and Yuan, L. C., <i>Phys. Rev.</i> (In press), [$\sigma_T^{(-)}$; see above]
500 to 1500	(k)	Cool, R., Madansky, L., and Piccioni, O., <i>Phys. Rev.</i> (To be published.) [σ_T^+ and $\sigma_T^{(-)}$; transmission, C-CH ₂ and C-CD ₂ subtraction]

* This is not in any sense intended to be a complete list, but rather tends to emphasize recent work. Further references can be found in Ruderman, M. A., Henley, E. M. & Steinberger, J., *Ann. Rev. Nuclear Sci.*, **3** (1953).

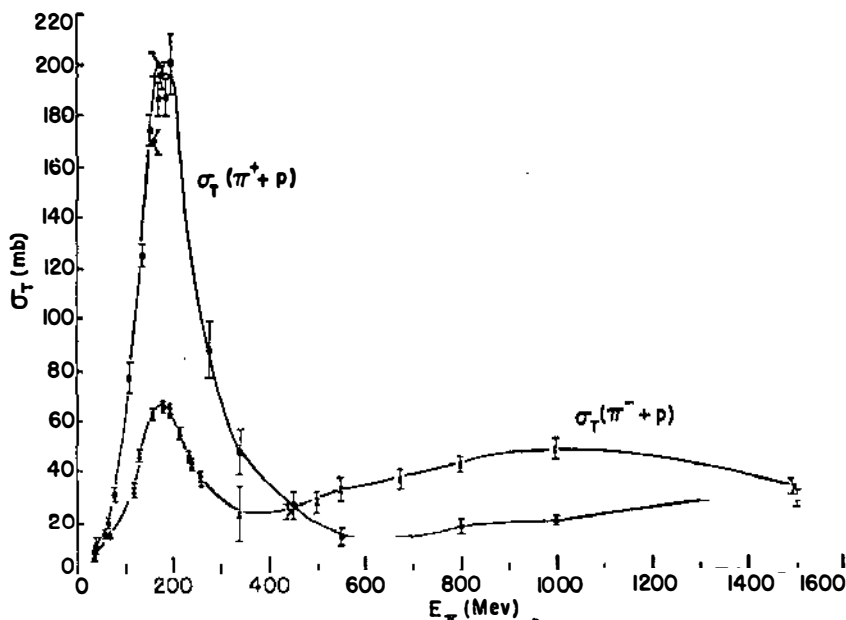


FIG. 3. Total cross section for π^+ -mesons and for π^- -mesons scattered by protons as a function of the meson energy in the laboratory system. The experimenters whose points are shown may be obtained from Table III by comparing energies. The π^+ points at 1.0 and 1.5 Bev. were obtained from $[\sigma(\pi^-, d) - \sigma(\pi^-, p)]$ —see reference (6) of Table III. The solid curves are drawn to fit as well as possible the experimental data.

$$\sigma_{1/2} = \frac{1}{2} [3\sigma_T^{(-)} - \sigma_T^{(+)}]. \quad 6.$$

It should be noted that these relations hold even when the cross sections are inelastic.

Using the experimental values for σ_T^{+} and $\sigma_T^{(-)}$ from Figure 3 we have obtained $\sigma_{3/2}$ and $\sigma_{1/2}$ as shown in Figure 4. The solid curves represent a fit to the points shown. The dashed curves represent the upper limit to the scattering in pure states of j and l . They are given by

$$\sigma(j, l) = 2\pi \left(\frac{\hbar}{q} \right)^2 [2j + 1] \quad 7.$$

where q is the momentum of the pion in the barycentric system. The presence of the peak at 200 Mev in $\sigma_{3/2}$ and its absence in $\sigma_{1/2}$ is quite striking and would seem to provide excellent evidence for the charge independence hypothesis as well as for Brueckner's (8) hypothesis of a resonance in the $(I=3/2, j=3/2, l=1)$ state. In particular, the peak height should be compared with the value of $\sigma(3/2, 1)$ as obtained from Equation (7).

If the peak in $\sigma_{1/2}$ at about one Bev were attributable to scattering in only one angular momentum state, this would require $j \approx 5$ [see also Fig. 1]. This

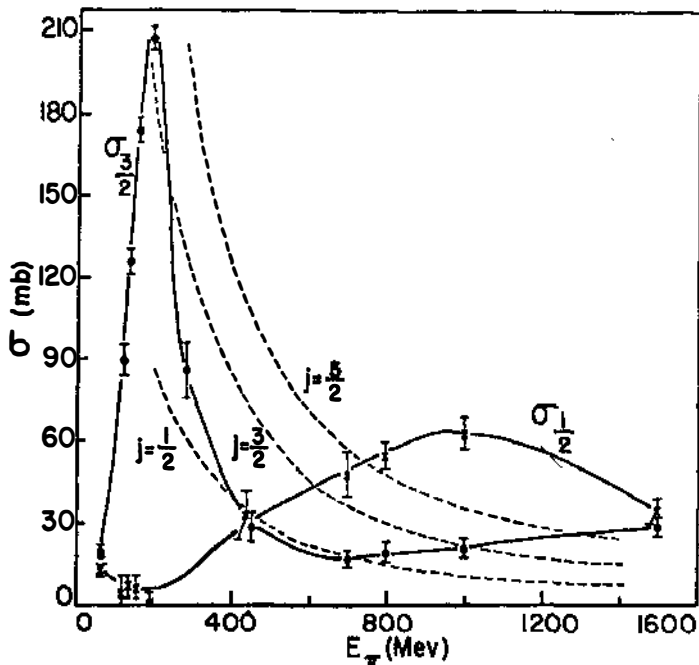


FIG. 4. The experimental values for the pion-nucleon scattering in the states of pure I -spin, $1/2$ and $3/2$. The dashed curves represent the upper limits on the cross sections for scattering in pure states (j, l) as given by Equation 7.

seems rather unlikely, especially since the cross section appears to be largely inelastic. On the other hand, this peak does perhaps suggest the possibility that at one Bev the inelastic (*i.e.*, meson-production) cross section may be predominantly in the $I=1/2$ state. If this were true, charge independence gives the relations:

$$\begin{aligned} \sigma(p + \pi^- \rightarrow n + \pi^- + \pi^+) + \sigma(p + \pi^- \rightarrow n + \pi^+ + \pi^-) \\ = 2\sigma(p + \pi^- \rightarrow n + \pi^0 + \pi^0) + \sigma(p + \pi^- \rightarrow p + \pi^- + \pi^0) \end{aligned} \quad 8.$$

and

$$\sigma(p + \pi^- \rightarrow p + \pi^0 + \pi^-) = \sigma(p + \pi^- \rightarrow p + \pi^- + \pi^0).$$

These are differential cross sections. For the total cross sections we have

$$\begin{aligned} 2\sigma_T(p + \pi^- \rightarrow n + \pi^- + \pi^+) = \sigma_T(p + \pi^- \rightarrow n + \pi^0 + \pi^0) \\ + \sigma_T(p + \pi^- \rightarrow p + \pi^- + \pi^0). \end{aligned} \quad 9.$$

These relations will be valid only when (and if) the production cross sections for π^+ -mesons striking protons are small, as stated above.

The smallness of the $I=1/2$ scattering below 400 Mev has been noted by Ashkin (10). The Carnegie Tech² data shows this rather strikingly if we plot $3\sigma_T^{(-)}$ and σ_T^{+} to the same scale, as is done in Figure 5.

Brueckner (8) proposed fitting the ($j=3/2$, $l=1$, $I=3/2$) state to a one-level resonance formula. Let us suppose that the entire $\sigma_{T^+} = \sigma_{3/2}$ cross section arises from this state. Then

$$\sigma_{T^+} = \frac{2\pi\hbar^2}{q^2} \frac{\Gamma^2}{(E - E_0)^2 + \frac{\Gamma^2}{4}}, \quad (10)$$

where

$$\Gamma = \left[\frac{2 \left(\frac{qa}{\hbar} \right)^3}{1 + \left(\frac{qa}{\hbar} \right)^2} \right] \gamma_\lambda^2. \quad (11)$$

Here E_0 is the "resonance energy" in the barycentric system, γ_λ^2 is the reduced width, and a is the channel radius. E is the energy of the meson and nucleon in the barycentric system. We have chosen

$$\begin{aligned} E_0 &= 159 \text{ mev} \\ \gamma_\lambda^2 &= 58 \text{ mev} \\ a &= 0.88 \left(\frac{\hbar}{\mu c} \right). \end{aligned} \quad (12)$$

The resulting value of σ^+ as calculated from Equation 10 is compared with the experimental cross sections in Figure 5. The fit is evidently excellent.

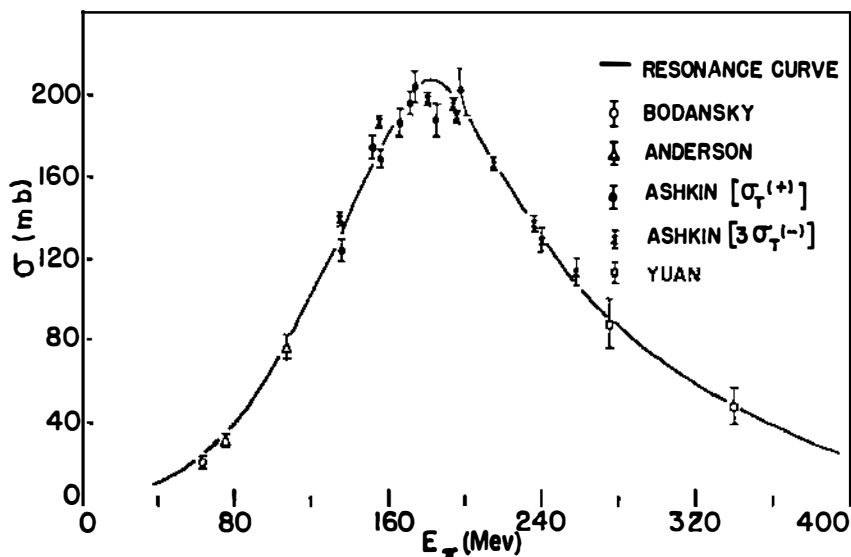


FIG. 5. A one-level resonance formula for the σ^+ scattering as calculated from Equation 10 is compared to the experimental cross sections. The references are given in Table III. The points ($\frac{3}{2}$) are $3\sigma_T^{(-)}$, as measured by Ashkin, *et al.* (10).

Angular distributions for pion scattering are also available (see the references of Table III). Here the experimental information is much less complete than for the total cross sections. Were the scattering entirely in the ($I=3/2$, $j=3/2$, $l=1$) state all the angular distributions would be of the form

$$(1 + 3 \cos^2 \theta), \quad 13.$$

where θ is the scattering angle in the barycentric system. Some measured angular distributions for σ^+ are presented in Figure 6. The asymmetry about 90° is evidently incompatible with the expression 13. On the other hand, a

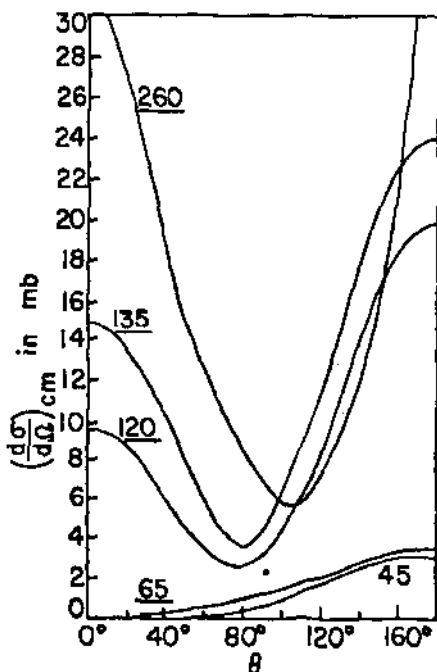


FIG. 6. Some angular distributions in the barycentric system for the scattering $\pi^+ + p \rightarrow \pi^+ + p$. The meson energies in the laboratory system are indicated in Mev. No attempt is made to indicate the experimental errors. The 45 Mev curve is that of Orear, J., Lord, J., and Weaver, A., *Phys. Rev.* **93**, 575 (1954); the 65 Mev curve is taken from reference (d) of Table III; the 120 and 135 Mev cross sections are those of reference (e) of Table III; the 260 Mev curve is that of Fowler, W. B., Lea, R., Shepard, W. D., Shutt, R. P., Thorndike, A. M., and Whittemore, W. L., *Phys. Rev.* **92**, 832 (1953).

relatively small admixture of S -state scattering can lead to the observed asymmetry. The general expression for a cross section with only S - and P -wave scattering is

$$\sigma = a + b \cos \theta + c \cos^2 \theta \quad 14.$$

which seems to be compatible with available pion scattering angular distributions. Indeed, it appears possible to fit the presently known angular distribution with P -wave scattering only in the ($I=3/2$, $j=3/2$) state and with an admixture of S -wave scattering (11).

A great deal of effort has been expended in trying to determine the phase shifts for pion-nucleon scattering. According to charge independence there should be a total of six for energies such that only S - and P -waves contribute. Following the notation of Anderson *et al.*⁷ we shall designate the S -wave phase shifts as α_3 and α_1 for the $I=3/2$ and $1/2$ states, respectively. The four P -wave phase shifts are designated by α_{33} , α_{13} , α_{31} , and α_{11} , where the first index is twice I and the second is twice j . The determination of these six phase shifts from the experimental data is very ambiguous at present and various solutions have been obtained (12), so we prefer not to engage in a detailed discussion.

The simplest set of phase shifts (and from the present theoretical point of view perhaps the most reasonable) is that of Bethe [with de Hoffmann *et al.* (see 11)], for which α_{13} , α_{31} , and α_{11} are small and α_{33} passes through

TABLE IV
MESON-NUCLEON SCATTERING PHASE SHIFTS*

E_{π} Mev	α_{33}	α_3	α_1
120	30°	-12°	8°
217	107°	-20°	-4°

For $E_{\pi} < 120$ Mev,

$$\alpha_{33} \cong 16^\circ \left(\frac{q}{\mu c} \right)^3$$

* Values of Bethe *et al.* (11) are given for the pion-nucleon scattering phase shifts α_{33} , α_3 and α_1 ; α_{33} and α_3 can be extrapolated linearly in the energy range from 120 to 217 Mev. α_1 should be extrapolated as a parabola with zero slope at 120 Mev in this energy range.

90° for the meson energy $E_{\pi} = 195$ Mev (in the laboratory system). Values for these phase shifts are given in Table IV. Some arguments in favor of this set are:

(a). The very good fit of $\sigma_{3/2}$ to the resonance formula 10 [see Fig. 5]. Were α_{33} not to pass through 90° it would probably be necessary to have a peak in at least two phase shifts at $E_{\pi} \cong 200$ Mev (11). For this argument not only the shape, but also the magnitude of the cross section is important [see also Fig. 4].

(b). The coefficient b in equation 14 changes sign in the vicinity of $E \cong 180$

⁷ See Table III, reference (e).

Mev [see Fig. 6]. This is certainly compatible with an α_{33} which passes through 90° at about this energy.

(c). This behavior is strongly suggested by meson theory and is suggestive of the reason for the lack of a peak in $\sigma_{1/2}$ at 300 Mev [see Fig. 4].

(d). The energy dependence and the change in sign of the interference term for π^+ -photoproduction (see the section on PHOTOPRODUCTION OF π -MESONS FROM NUCLEONS) strongly suggest "resonant" scattering.

Some further discussion of the scattering phase shifts along with their interpretation is given in the section on MESON THEORY.

THE PHOTOPRODUCTION OF π -MESONS FROM NUCLEONS

Here there are four photo-processes which can occur (along with their inverses):

$$\gamma + p \rightarrow \pi^+ + n \quad (P_{\gamma^+})$$

$$\gamma + n \rightarrow \pi^- + p \quad (P_{\gamma^-})$$

$$\gamma + p \rightarrow \pi^0 + p \quad (P_{\gamma^0})$$

$$\gamma + n \rightarrow \pi^0 + n. \quad (P_{\gamma^{n0}})$$

The reactions P_{γ^-} and $P_{\gamma^{(n0)}}$ must be studied using bound neutrons (preferably in deuterium), so their measurement is more difficult and uncertain.

We shall tentatively assume that for E_γ (the γ -ray energy in the laboratory system) < 300 to 400 Mev the meson is emitted with appreciable probability only into S - and P -states with respect to the nucleon. The arguments in favor of this point of view were described in the section, SOME PHYSICAL CONCEPTS APPLYING TO PIONS. From Figure 2 we concluded that the process P_{γ^0} involved mostly P -state emission, whereas P_{γ^+} involved an admixture of S - and P -states at "low energies." The differential cross sections will then have the general form (in the barycentric system)

$$\sigma = A_0 + A_1 \cos \theta + A_2 \cos^2 \theta. \quad 15.$$

We shall designate the individual differential cross sections for the four reactions listed above as

$$\sigma(\gamma^+), \sigma(\gamma^-), \sigma(\gamma^0), \text{ and } \sigma(\gamma^{n0}),$$

respectively. The total cross sections will be written as $\sigma_T(\gamma^+)$, etc. The quantity A_0 contains in general contributions from both S - and P -waves and can thus be written as

$$A_0 = A_0(S) + A_0(P), \quad 16.$$

where $A_0(S)$ and $A_0(P)$ refer respectively to S - and P -states. The energy dependence of the A 's may be obtained from Table I at "low energies." (We tentatively take this to mean that $E_\gamma < 250$ Mev. The upper limit for "low energy" is not, of course, known *a priori*.) For such energies we write

$$\begin{aligned} A_0(S) &= \frac{\eta}{\nu} g_{0S} \\ A_0(P) &= \eta^2 \nu g_{0P} \\ A_1 &= -\eta^2 g_1 \\ A_2 &= -\eta^2 \nu g_2, \end{aligned} \quad 17.$$

where η is the momentum of the meson and ν is that of the photon in units of μc in the barycentric system. This dependence on ν is not determined from general considerations but is a guess based on meson theory. For energies sufficiently close to the energetic threshold the actual ν dependence is not important.

Cross sections for P_{γ^0} at about 90° in the laboratory frame of reference were measured by Silverman & Stearns (13) at Cornell, who observed the energy and angle of the recoil proton for E_γ between 200 and 300 Mev. A similar study has been made at Massachusetts Institute of Technology by Osborne *et al.* (14), who included an analysis of the angular distribution; some of these data are shown in Figure 2. At California Institute of Technology the cross section for P_{γ^0} has been studied for several angles and values of E_γ between 270 and 450 Mev. This work has been done by Walker, Oakley & Tollestrop (15), and more recently some further preliminary information has been obtained by these workers.⁸ The method used was similar to that of the Cornell group, the pulse height and range of the recoil protons being observed in coincidence with a decay γ -ray of the π^0 .

The differential cross sections for P_{γ^+} have recently been studied by Bernardini & Goldwasser (16)⁹ for $E_\gamma < 200$ Mev. The cross sections at low energies are evidently of particular importance for the determination of the g 's of Equation 17 (and the multipole moments, as discussed in the part on *The angular distributions* of this section).

At CIT² the reaction P_{γ^+} has been studied for E_γ between 200 and 400 Mev. Tollestrop, Keck & Worlock (17) have used a scintillation counter telescope to measure ionization versus residual range for the π^+ -mesons. Walker *et al.* (18) have measured the same cross sections, determining the pion energy and angle by a magnetic spectrometer. The CIT data concerning P_{γ^+} have been analyzed in the form of Equation 15 by Bacher *et al.* (19).¹⁰

The total cross sections.—We consider first the total cross sections for photomeson production. At sufficiently low energies the meson should be emitted into an S -state; however, as noted above, the S -wave amplitude seems to be quite small for π^0 -production. This may be qualitatively understood on the basis of a simple model. Emission of a pseudoscalar pion into an S -state must occur only via electric dipole absorption of the photon (20, 21). If we suppose the amplitude for this to be proportional to the static electric dipole moment (with respect to the center-of-mass) for the appropriate meson-nucleon system in the final state, then

⁸ Personal communication. We are much indebted to Professor Walker for informing us of this work.

⁹ We are deeply indebted to Professors Bernardini and Goldwasser for discussions in advance of publication of their work.

¹⁰ We should emphasize that the π^+ data and their analysis are still somewhat preliminary. We are greatly indebted to the CIT group for permission to quote their work.

$$\sigma(\gamma^-):\sigma(\gamma^+):\sigma(\gamma^0):\sigma(\gamma^{n0}) = [1 + \mu/M]^2:1:1/2\left(\frac{\mu}{M}\right)^2:0 \quad 18.$$

as long as only S -waves need be considered; here, M is the nucleonic mass.

This is in agreement with the observed smallness of the S -wave term for π^0 -production. Also

$$\frac{\sigma(\gamma^-)}{\sigma(\gamma^+)} = [1 + \mu/M]^2 = 1.32, \quad 19.$$

on the basis of Equation 18. This is compatible with recent measurements by Sands *et al.* (22) which appear to give

$$\frac{\sigma(\gamma^-)}{\sigma(\gamma^+)} \approx 1.4 \quad 20.$$

for very low energy pions.

In view of the successful application of the one-level resonance formula of Equation 10 to the scattering (Fig. 5) it is natural to try the same approach here, considering photoproduction as the reaction channel of the scattering. To the extent that emission of the pion into the ($I=3/2, j=3/2$) state is predominant, this should be a satisfactory approximation for energies near the "resonance" which occurs for $E_\gamma \approx 340$ Mev. Then (13, 23) (κ is the photon momentum in the barycentric system)

$$\sigma_T(\gamma^0) = 2\pi \left(\frac{\hbar}{\kappa}\right)^2 \frac{\Gamma_\gamma \Gamma}{(E - E_0)^2 + \frac{\Gamma^2}{4}}, \quad 21.$$

where Γ , E , and E_0 were defined in connection with Equation 10. For the reaction width Γ_γ we have [$\nu = \kappa/\mu c$]

$$\Gamma_\gamma = \frac{\nu^2 f_\gamma}{1 + \left(\frac{a\kappa}{\hbar}\right)^2} \quad 22.$$

with the channel radius a given by Equation 12. Here f_γ plays the role of a reduced width and is the only arbitrary parameter in Equation 21. We choose f_γ to be constant and to have the value

$$f_\gamma = 0.10 \text{ Mev.} \quad 23.$$

In Figure 7 $\sigma_T(\gamma^0)$ as calculated from Equation 21 is compared with the experimental cross sections. [The "experimental" values of σ_T were obtained from the observed differential cross sections on the assumption that the angular distribution is of the form of expression 39 (see *The angular distributions* in this section).] Accepting our determination of $\sigma_T(\gamma^0)$ from the experimental data, one can hardly find any fault with the fit of Equation 21 to this for $0 < E_\gamma < 450$ Mev. The possible validity of Equation 21 is especially interesting in view of the fact that f_γ was our only free parameter.

Let us now investigate the total cross section for P_γ^+ ; i.e., $\sigma_T(\gamma^+)$. Here the low energy cross sections indicate that an S -wave term is required (see

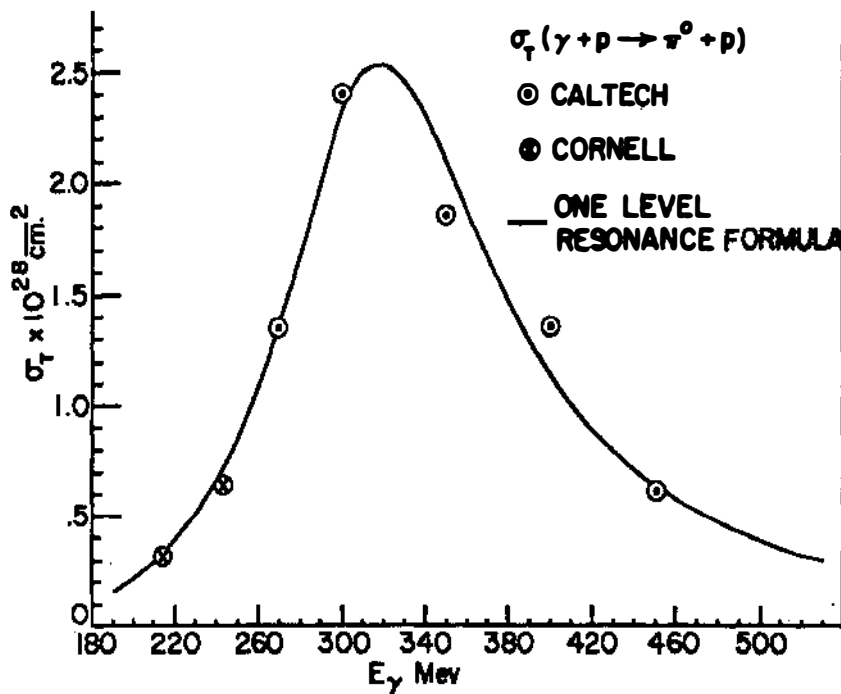


FIG. 7. A comparison of $\sigma_T(\gamma + p \rightarrow \pi^0 + n)$ as calculated using the "resonance equation" 21 with experiment. The first two points are those of Equations 17 and the remainder are from Equations 19 and 20.

Fig. 2 and reference 16). We shall assume that the only modification of Equation 21 is in the appearance of an S -wave term as given by pseudo-scalar meson theory [except that the P -wave contribution to $\sigma_T(\gamma^+)$ is only one half as great as it is for $\sigma_T(\gamma^0)$ by charge independence (20)]. Then

$$\sigma_T(\gamma^+) = \sigma_0 \left(\frac{q}{\kappa} \right) \left[\frac{1 - \frac{\kappa}{Mc}}{\left(1 + \frac{\kappa}{Mc} \right)^2} \right] + \frac{1}{2} \sigma_T(\gamma^0). \quad 24.$$

As mentioned above, the first term has the energy dependence expected from meson theory. We choose σ_0 to be constant and equal to

$$\sigma_0 = 2.5 (10)^{-28} \text{ cm}^2. \quad 25.$$

In Figure 8 we compare Equation 24 with the experimental values for $\sigma_T(\gamma^+)$. The agreement is clearly not as good as it seemed to be for $\sigma_T(\gamma^0)$. The calculated curve does not fall off rapidly enough at high energies, but this may very well be the fault of the S -wave term in Equation 32, which by itself is larger than the experimental cross sections above 400 Mev. Also the

peak in the calculated curve seems to come at somewhat too high an energy. When we discuss the angular distribution in the following part it will become apparent that Equations 21 and 24 are oversimplifications. On the other hand, the agreement of these equations with experiment, as evidenced in Figures 7 and 8, is fairly good. Consequently, it is not at all unlikely that (except for finer details) the ($I=3/2, j=3/2$) state is quite significant for photoproduction and indeed Equations 21 and 24 may be reasonable approximations to the actual cross sections.

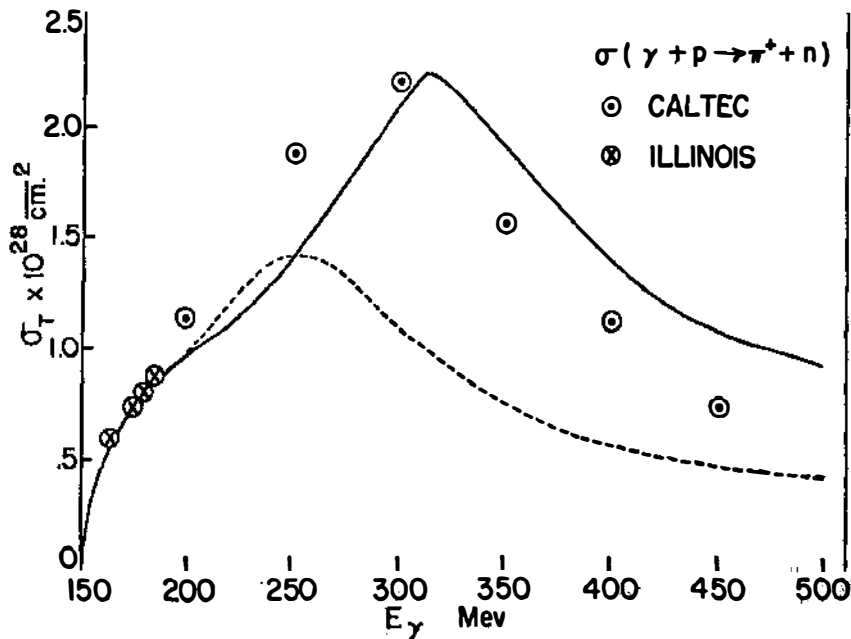


FIG. 8. A comparison of $\sigma_T(\gamma + p \rightarrow \pi^+ + n)$ as calculated from Equation 32 with experiment. The first three points are those of reference (16); the remainder are from references (17), (18) and (19). The dotted curve represents $[\sigma_T(\gamma^+) - \frac{1}{2}\sigma_T(\gamma^0)]$.

There is evidently no *a priori* justification for choosing the γ -ray channel radius a in Equation 22 the same as that for the meson (Equation 11). Indeed, in Equation 17 we have taken this radius for the γ -ray equal to zero. The finite value used in Equation 22 was of some help in bringing the cross sections down rapidly above the resonance peak. We do not, of course, even know that f_γ should remain constant over an appreciable energy interval, so it is not possible at present to say very much about the κ -dependence of the cross sections (except that it should not be important at "low energies").

We must also observe that the term "resonance" as applied to the scattering and photoproduction has not been precisely defined by our discussion. Aside from the statement that α_{33} passes through 90° for $\epsilon_\gamma \approx 195 \text{ Mev}$,

one would desire its energy dependence in this energy region. The Equations 10 and 21 are based on the analogy to resonant nuclear reactions, but probably cannot be justified in detail at relativistic energies (24). The most satisfactory study of resonant reactions involving relativistic pions seems to be that recently proposed by Sachs (24). His expression for a "resonant" cross section is rather similar to Equation 10.

The angular distributions.—The angular distribution for the cross section $\sigma(\gamma^+)$ has been analyzed in the form of Equation 15 by Bacher *et al.* (19) for

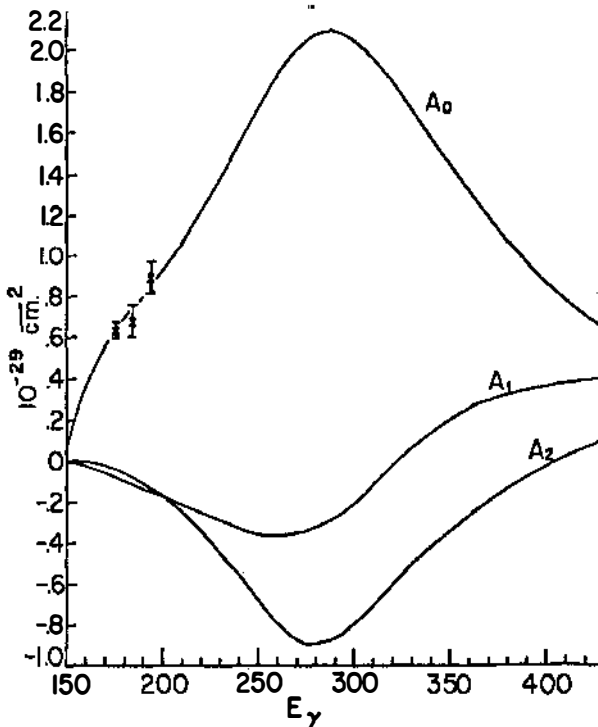


FIG. 9. Experimental values of the coefficients A_0 , A_1 , and A_2 in the angular distribution for $\gamma + p \rightarrow \pi^+ + n$. (See Equation 21.) The three indicated points are taken from reference (16); the curves above 250 Mev. are those of reference (19).

E_γ between 250 and 450 Mev. A similar analysis has been made by Bernardini & Goldwasser (25) for $E_\gamma < 250$ Mev. The resulting coefficients in Equation 15 are plotted versus energy in Figure 9. Our coefficients in Figure 9 actually differ somewhat from those of the above references in that we have imposed on them the energy dependence implied by Equations 17 for $E_\gamma < 250$ Mev. This change is compatible with present experimental uncertainties. The values which we have used for the coefficients g of Equations 17 are given in Table V.

TABLE V

THE COEFFICIENTS g OF EQUATIONS 17 FOR THE ANGULAR DISTRIBUTION FOR PHOTOMESON PRODUCTION*†

Process	$\gamma + p \rightarrow \pi^+ + n$	$\gamma + p \rightarrow \pi^0 + p$
$g_{\pi\pi}$	10.5	—
$g_{\pi\rho}$	4.0	8.5
g_1	3.0	—
g_2	3.5	7.3

* The g 's are given in units of 10^{-30} cm². $g_{\pi\pi}$ is probably accurate to within 10 per cent. The remaining g 's may be in error by as much as 25 per cent, although their relative values are probably much more accurate.

† The π^0 data are those of Oakley, D., and Walker, R. L. (24a) and of Osborne *et al.* (26).

The available information concerning the angular distribution $\sigma(\gamma^0)$ is still meager. In Table VI we give the values deduced by Osborne *et al.* (26) for the coefficients of Equation 15. Values for the $g_{\pi\rho}$ and g_2 of Equations 17, which have been obtained from this and the work at Caltech, are included in Table V.

Perhaps the most striking conclusion from Table V is that the P -wave contributions to $\sigma(\gamma^0)$ and $\sigma(\gamma^+)$ appear to have a similar angular dependence

TABLE VI

THE ANGULAR DISTRIBUTION FOR THE REACTION $\gamma + p \rightarrow \pi^0 + p^*$

γ -ray energy	220 to 280 Mev	280 to 330 Mev
A_0	9 ± 1	18 ± 1
A_1	-2.5 ± 1	2.3 ± 1
A_2	-7.5 ± 2	-15 ± 3

* The coefficients A_0 , A_1 , and A_2 are those of Equation 31. This is based on preliminary MIT data.† The units are 10^{-30} cm²/steradian.

† We are much indebted to Dr. Osborne for several discussions of this work, which is still quite preliminary. See reference (26).

and show a ratio of approximately 2:1 in strength. This is in agreement with the simple "resonance" theory given in the first part, *The total cross sections*, of this section. We proceed now with a more precise development of this theory.

A quantitative discussion of the angular distribution for photoproduction may conveniently be given in terms of a multipole expansion of the amplitudes for γ -ray absorption (25, 26). Because of the pseudoscalar parity of the pion, if the meson is emitted into an S -state, the transition must

be electric dipole with matrix element E_1 . If the meson is emitted into a P -state with $j=1/2$, then the transition must be magnetic dipole with matrix element $M_1(1/2)$. A P -state with $j=3/2$ may be either magnetic dipole with matrix element $M_1(3/2)$ or electric quadrupole with matrix element E_2 . Since an arbitrary combination of these may occur, we can write the scattering amplitude T for photoproduction in the concise form (20) (in the barycentric system):

$$\begin{aligned} T = & iE_1 \delta \cdot \hat{\epsilon} - M_1(1/2) [\boldsymbol{\kappa} \times \hat{\epsilon} \cdot \mathbf{q} - i \delta \cdot (\boldsymbol{\kappa} \times \hat{\epsilon}) \times \mathbf{q}] \kappa^{-1} q^{-1} \\ & - M_1(3/2) [2 \boldsymbol{\kappa} \times \hat{\epsilon} \cdot \mathbf{q} + i \delta \cdot (\boldsymbol{\kappa} \times \hat{\epsilon}) \times \mathbf{q}] \kappa^{-1} q^{-1} \\ & + iE_2 1/2 [\sigma \cdot \boldsymbol{\kappa} \hat{\epsilon} \cdot \mathbf{q} + \delta \cdot \hat{\epsilon} \boldsymbol{\kappa} \cdot \mathbf{q}] \kappa^{-1} q^{-1}. \end{aligned} \quad 26.$$

Here δ is the nucleon spin, $\boldsymbol{\kappa}$ is the photon momentum and $\hat{\epsilon}$ is its polarization vector, and \mathbf{q} is the momentum of the meson. As mentioned above, there are four photoproduction processes, so we have actually four T 's to be designated by

$$T^+, T^-, T^0, T(n0).$$

Also, of course, there will be four sets of each of the multipole moments: E_1^+ , E_1^- , etc.

The differential cross section σ is obtained in the usual manner by averaging $|T|^2$ over spin and polarization states:

$$\begin{aligned} \sigma = W \{ & |E_1|^2 + |M_1(1/2)|^2 + |M_1(3/2)|^2 1/2 [5 - 3 \cos^2 \theta] \\ & + |E_2|^2 1/8 [1 + \cos^2 \theta] \\ & - 2 \operatorname{Re} [E_1^* (M_1(3/2) - M_1(1/2)) - 1/2 E_2] \cos \theta \\ & - 1/2 \operatorname{Re} [E_2^* (M_1(3/2) - M_1(1/2))] [3 \cos^2 \theta - 1] \\ & - \operatorname{Re} [M_1^*(3/2) M_1(1/2)] [3 \cos^2 \theta - 1] \}. \end{aligned}$$

In this equation θ is the angle between $\boldsymbol{\kappa}$ and \mathbf{q} (all quantities referred to the barycentric system, as stated above). " $\operatorname{Re} (\dots)$ " means the "real part of (\dots) "; W is the statistical weighting factor, which is approximately

$$W = (2\pi)^4 \frac{\eta \omega}{\left[1 + \frac{\kappa}{Mc}\right]^2}, \quad 28.$$

where $\omega = [1 + \eta^2]^{1/2}$ and M is the nucleonic mass.

We have, of course, four cross sections, $\sigma(\gamma^+)$, $\sigma(\gamma^-)$, etc. of the form of Equation 27. This means that there are 16 multipole amplitudes. When use is made of charge independence (7), then this number is reduced to 12 independent multipole amplitudes. Although these are complex quantities, this complexity is rather trivial. Indeed, in an appropriate representation the complex phases of the multipole amplitudes can be explicitly calculated in terms of the six phase shifts α (see SCATTERING OF PIONS BY NUCLEONS) characterizing the pion scattering.¹¹ This leaves us with twelve

¹¹ This fact has been noted independently by K. Aizu, by E. Fermi (Unpublished data), and by K. Watson (27a).

"real" parameters to describe the four photo cross sections. These arguments as well as the explicit form of the amplitudes are given in the Appendix.

The coefficients A_0 , A_1 , and A_2 of Equation 15 can be expressed in terms of the multipole amplitudes by means of Equation 27. We do this in a form suggested by Fermi (27). Let us define

$$\begin{aligned} X &\equiv 3/2M_1(3/2) + 1/4E_2 \\ Y &\equiv 1/2[M_1(3/2) - 1/2E_2] + M_1(1/2) \\ K &\equiv [M_1(3/2) - 1/2E_2] - M_1(1/2). \end{aligned} \quad 29.$$

Then

$$\begin{aligned} A_0 &= W[|E_1|^2 + |X|^2 + |Y|^2] \\ A_2 &= W[|K|^2 - |X|^2 - |Y|^2] \\ A_1 &= -W[2 \operatorname{Re}[E_1^* K]]. \end{aligned} \quad 30.$$

From the expressions given in the Appendix for the multipole matrix elements, we see that these are all "real" for energies sufficiently near threshold that the scattering phase shifts are small [say $E_\gamma < 250$ Mev for $\sigma(\gamma^+)$]. This is the energy range for which we have assumed that Equations 17 remain valid. For these energies, then E_1 , X , Y , and K are "real" and are expected to have a simple energy dependence. We obtain, then immediately from Equations 30 a useful relation due to Fermi (27) between the g 's of Equations 17:

$$g_1^2 = 4g_{0S}[g_{0P} - g_2]. \quad 31.$$

Since the four g 's can in principle each be determined experimentally, Equation 31 presents a test of the present model of photoproduction. It is satisfied to within the experimental errors by the values given in Table V for $\sigma(\gamma^+)$.

The small value of g_1^+ in Table V shows that $\sigma(\gamma^+)$ varies approximately as

$$\sin^2 \theta$$

by Equation 31. We can determine directly from g_{0S}^+ the strength of the electric dipole term near threshold. This is

$$\sqrt{\omega} E_1^+ = \sqrt{\frac{\eta}{\nu}} 3.3(10)^{-16} \text{ cm.} \quad 32.$$

The smallness of g_1^+ makes it impossible to fit the threshold cross sections with a single P -wave multipole term, in contrast to the Brueckner-Watson hypothesis (20). The data may be fitted, however, with any two of the three terms, $M_1(3/2)$, $M_1(1/2)$, or E_2 , nonvanishing. There is no reason to suppose that all three are not present.

Because of the smallness of E_1^0 near threshold for $\sigma(\gamma^0)$, the relation 31 is not expected to be valid for any appreciable energy interval for π^0 -cross sections. Here a more elaborate analysis is required (see footnote 11).

To summarize our arguments for $\sigma(\gamma^+)$ and $\sigma(\gamma^0)$ near threshold, we find

an appreciable electric dipole term only for the former. The P -wave contributions to both cross sections appear to vary with angle roughly as $\sin^2 \theta$. The 2:1 ratio of P -waves implied by the "resonance theory" seems to hold fairly well.

Let us now investigate the experimental data in the resonance region. Since the resonant ($I=3/2$, $j=3/2$, $l=1$) state can contribute only to $M_1(3/2)$ and E_2 , we can expect these terms to be predominant. The expected enhancement of these two terms in the resonance region does not, of course, imply that the other multipole terms will be absent. However, if the resonance theory is valid, we should be able to neglect the remaining terms in a first approximation. Then, from the form given in the Appendix for the multipole moments, we can write (near the resonance energy)

$$\begin{aligned}\sqrt{W} M_1(3/2) &= e^{i\alpha_1} \sin \alpha_{33} \left[\frac{\nu^{1/2}}{\eta^{3/2}} \right] \mu_1 \\ \sqrt{W} E_2 &= e^{i\alpha_2} \sin \alpha_{33} \left[\frac{\nu^{1/2}}{\eta^{3/2}} \right] \epsilon_2.\end{aligned}\quad 33.$$

Here μ_1 and ϵ_2 are real constant amplitudes. The complex factors $e^{i\alpha_{33}}$ appear for the reasons given in the Appendix. The factor $\sin \alpha_{33}$ represents a generalization of the energy dependence of Equation 21. Its use was discussed in reference (23).¹²

For the study of $\sigma(\gamma^+)$, Figure 9 shows that E_1^+ is also of importance. From the form given in the Appendix for E_1^+ and E_1^0 and also from the observation that E_1^0 seems to be very small at the energetic threshold, we shall set [in the notation of the Appendix]

$$2E_1^{(3)} = 1/2[E_1^{(1)} - 2\delta E_1^{(1)}]. \quad 34.$$

This assures us that $E_1^0=0$ at threshold, which seems to be a reasonable approximation. Using Equation 34, we can write E_1^+ as

$$\sqrt{W} E_1^+ = \sqrt{\frac{\eta}{\nu}} \frac{\epsilon_1}{3} [e^{i\alpha_1} + 2e^{i\alpha_1}]. \quad 35.$$

Here ϵ_1 has been defined by

$$\sqrt{2W} E_1^{(3)} = \sqrt{\frac{3}{\nu}} \epsilon_1.$$

We shall further suppose that ϵ_1 may be taken as a constant for $E_\gamma < 300$ Mev (which of course may not be correct).

Equations 30 may now be written as [using Equations 33 and 35]

¹² Equation 33 puts the resonance theory on a considerably sounder basis than did Equation 21, but are equivalent to this equation if α_{33} is of the form implied by Equation 10. For instance the more sophisticated theories of Sachs (29) and of Chew (see the section on MESON THEORY) will both relate the photoproduction to the scattering in the resonance region by equations 33.

$$\begin{aligned}
 A_1^+ &= -2/3 \frac{\sin \alpha_{33}}{\eta} \epsilon_1 [\mu_1 - 1/2\epsilon_2] [\cos(\alpha_{33} - \alpha_3) + 2 \cos(\alpha_{33} - \alpha_1)] \\
 A_2^+ &= -\frac{\sin^2 \alpha_{33}}{4} \left[\frac{\nu}{\eta^3} \right] \{ [3\mu_1 + 1/2\epsilon_2]^2 - 3[\mu_1 - 1/2\epsilon_2]^2 \} \\
 A_0^+(P) &= \frac{\sin^2 \alpha_{33}}{4} \left[\frac{\nu}{\eta^3} \right] \{ [3\mu_1 + 1/2\epsilon_2]^2 + [\mu_1 - 1/2\epsilon_2]^2 \}.
 \end{aligned} \tag{36}$$

From Equation 32 we obtain

$$\epsilon_1 = 3.3(10)^{-15} \text{ cm.} \tag{37}$$

From the data of Figure 9 we can determine the three A 's and thus at any given energy Equations 36 represent three equations to determine two parameters: μ_1 and ϵ_2 . It is evident from Figure 8 that the energy dependence of the A 's may agree only roughly with that calculated from Equations 36.¹³

The first point of interest is that A_1^+ should change sign when $[\cos(\alpha_{33} - \alpha_3) + 2 \cos(\alpha_{33} - \alpha_1)] = 0$. Using the phase shifts obtained from Table IV, we find that this factor vanishes at $E_\gamma = 335$ Mev. The experimental value of A_1 , as obtained from Figure 9, vanishes at $E_\gamma \cong 325$ Mev. The agreement is evidently much better than the present experimental accuracy warrants. We may also use the slope of A_1^+ as it passes through zero to determine $(\mu_1 - 1/2\epsilon_2)$, since ϵ_1 is known. We obtain

$$\mu_1 - 1/2\epsilon_2 = 1.6(10)^{-15} \text{ cm.,} \tag{38}$$

although one must be cautious in view of the limited accuracy of the preliminary experiments quoted (19). We may now determine μ_1 and ϵ_2 by using either A_2^+ or $A_0^+(P)$. This leads to

$$\begin{aligned}
 \mu_1 &= 2.5(10)^{-15} \text{ cm.} \\
 \epsilon_2 &= 1.8(10)^{-15} \text{ cm.}
 \end{aligned} \tag{39}$$

Fortunately each of the two independent determinations leads to a value agreeing to within better than 10 per cent of those given by Equation 39. This seems to be a reassuring consistency check on our model (although perhaps not as impressive as its prediction of the vanishing of A_1^+ at $E_\gamma = 335$ Mev).

An important test of the resonance model is its prediction that

$$A_2^0/A_2^+ = \frac{A_0^0}{A_0^+(P)} = 2.$$

These relations do seem to be satisfied to within the experimental error (although the experimental ratio may be as large as 2.5). We also note [from Figure 9 and Table VI] that

¹³ It has been pointed out to us by Professors G. Chew and G. Bernardini that by giving up the energy dependence of α_{33} implied by Equation 10 and using instead that of Table IV, one can fit better both the scattering and photoproduction (using Equations 36).

$$\frac{A_0^+(P)}{A_2^+} \cong 1.35, \quad \frac{A_0^0}{A_2^0} \cong 1.3. \quad 40.$$

These are evidently subject to considerable experimental uncertainties ($\cong 25$ per cent). If there were no electric quadrupole contribution (20), this ratio should be $5/3 \cong 1.67$ in both cases. These ratios, as well as Equations 38 and 39 do suggest that there is an electric quadrupole contribution.

Having once determined the multipole amplitudes in the "resonance region" for P_7^+ the theory predicts uniquely the coefficient A_1^0 as well as A_0^0 and A_2^0 for P_7^0 . In the notation of Equations [see the Appendix and footnote 11]:

$$\begin{aligned} A_0^0 &\cong A_0^0(P) = 1/2 A_0^+(P) \\ A_2^0 &= 1/2 A_2^+ \\ A_1^0 &= -4/3 \frac{\sin \alpha_{33}}{\eta} \epsilon_1 [\mu_1 - 1/2 \epsilon_2] [\cos(\alpha_{33} - \alpha_3) - \cos(\alpha_{33} - \alpha_1)]. \end{aligned} \quad 41.$$

When more detailed experimental information is available, these equations will provide a critical test of the "resonance model."

Before attacking the more complicated phenomena of nucleonic production of mesons, we may summarize our conclusions from the scattering and photoproduction. The three postulates of the section on SOME PHYSICAL CONCEPTS APPLYING TO PIONS [i.e., (a) the finite range of interaction; (b) the hypothesis of charge independence; (c) the resonance hypothesis] have provided a rather successful and reasonably complete framework for describing the present evidence for meson phenomena at "low energies." This seems to imply a certain inherent simplicity in the phenomena and lessens the burden to be met by a more basic and detailed theory.

π -MESON PRODUCTION IN NUCLEON-NUCLEON COLLISIONS¹⁴

We shall consider in this section the emission and absorption of pions by a system of two nucleons. The emission process can be studied experimentally by observing pions produced in the bombardment of hydrogen with neutrons or protons (or in an equivalent experiment using polyethylene-carbon difference). Absorption of pions can be studied by means of the reactions $\pi^+ + D \rightarrow 2P$ and $\pi^- + D \rightarrow 2N$.

According to the hypothesis of charge independence, the isotopic spin I of two nucleons may undergo, during the emission of a pion, one of three changes:

$$\begin{aligned} I = 1 &\rightarrow I = 0, \text{ total cross section denoted by } \sigma_{10} \\ I = 0 &\rightarrow I = 1, \text{ total cross section denoted by } \sigma_{01} \\ I = 1 &\rightarrow I = 1, \text{ total cross section denoted by } \sigma_{11}. \end{aligned}$$

¹⁴ This section overlaps considerably in content a paper by A. H. Rosenfeld to be published in the *Physical Review*. His paper and this article were written at the same time with frequent consultation between him and the authors. We wish to thank Dr. Rosenfeld for many stimulating discussions.

The process $I=0 \rightarrow I=0$ is forbidden since the emitted pion carries away unit isotopic spin.

Since the deuteron has $I=0$, only the first process listed above can lead to deuteron formation. Accordingly we write $\sigma_{10} = \sigma_{10'} + \sigma_{10''}$, where $\sigma_{10'}$ refers to deuteron formation ("bound" reaction) and $\sigma_{10''}$ to the formation of two free nucleons with $I=0$ ("unbound" reaction).

The total cross sections for the various observable reactions involving meson production may be expressed in terms of $\sigma_{10'}$, $\sigma_{10''}$, σ_{01} , and σ_{11} as follows:

$$\begin{array}{ll}
 P + P \rightarrow \pi^+ + D & \sigma = \sigma_{10'} \\
 P + P \rightarrow \pi^+ + N + P & \sigma = \sigma_{10''} + \sigma_{11} \\
 P + P \rightarrow \pi^0 + P + P & \sigma = \sigma_{11} \\
 N + P \rightarrow \pi^0 + D & \sigma = 1/2\sigma_{10'} \\
 N + P \rightarrow \pi^0 + N + P & \sigma = 1/2\sigma_{10''} + 1/2\sigma_{01} \\
 N + P \rightarrow \pi^+ + N + N & \sigma = 1/2\sigma_{11} + 1/2\sigma_{01} \\
 N + P \rightarrow \pi^- + P + P & \sigma = 1/2\sigma_{11} + 1/2\sigma_{01}.
 \end{array} \tag{42}$$

The factor $1/2$ occurs in the N - P cross sections because the N - P system has equal probabilities of having $I=1$ and $I=0$, while the P - P system always has $I=1$.

The two observable absorption reactions may be expressed in terms of $\sigma_{10'}$ by means of the principle of detailed balancing:

$$\begin{array}{ll}
 \pi^+ + D \rightarrow P + P & \\
 \pi^- + D \rightarrow N + N & \sigma = \sigma_{10'} \cdot 2/3 \frac{p^2}{\mu^2 c^2} \frac{1}{\eta^2}.
 \end{array} \tag{43}$$

Here μ is the pion mass, c the velocity of light, p the final momentum of each neutron in the center-of-mass system, and η the pion momentum in the center-of-mass system in units of μc .

All the relations based on charge independence are subject to correction on account of Coulomb forces, the mass difference between neutron and proton, the mass difference between charged and neutral pion, and other minor charge-dependent effects, apart from any gross failure of the principle of charge independence.

We shall restrict our discussion of meson production to a region of bombarding energy (<450 Mev) in which η is always <1 . In the same energy interval the internal energy E of the residual two-nucleon system is always $<\mu c^2(\sqrt{2}-1) = 57$ Mev and tends to be much lower because of a preference for high pion momenta, as we shall see; let us say $E < 25$ Mev for the most part. Now we may imagine the phenomenon of meson production to take place at a characteristic distance R from the center of collision, and R is certainly of the order of magnitude of $\hbar/\mu c$. In the energy region we are considering the product of R by either final neutron momentum or final meson momentum is thus $\leq \hbar$. We may safely suppose, then, that the meson will be emitted in an S - or a P -state with respect to the two nucleons and that the two final nucleons will be in an S - or a P -state relative to each other. In

the case of the nucleons the strong attractive force in the S -state (as contrasted with the relatively weak forces in the P -state at low energies) will strongly favor $l=0$ as well as enhancing the preference for low values of E . In the case of the meson, experiment reveals that the P -state is preferred (except very close to threshold), which indicates that the coupling responsible for meson production is primarily an interaction of the nucleon spin with the meson momentum, as in pseudoscalar meson theory.

In the absorption experiments, the internal angular momentum of the initial two nucleon system is always that of the deuteron, which is preponderantly 3S_1 with a small (~ 4 per cent) admixture of 3D_1 . Again we shall be concerned only with energies for which $\eta < 1$ and so the principal contribution to the process will come from S - and P -state mesons.

Let us consider first the best-known reaction, which is the one involving the formation (or disintegration) of a deuteron. The cross section for meson production with deuteron formation is the one we have designated by σ_{10} . In this process, if the meson is emitted in an S -state, the total angular momentum of the final state is merely that of the deuteron, $J=1$. The parity in the final state is negative for a pseudoscalar meson. In the initial state of the two nucleons, then, we must have $I=1$, $J=1$, and negative parity. According to the Pauli principle a two-nucleon wave function symmetric in isotopic spin ($I=1$) and antisymmetric in space (negative parity) must be symmetric in spin (triplet). Thus the only possible initial configuration is 3P_1 .

If the meson is emitted in a P -state, the total angular momentum of the final state may be $J=0, 1$, or 2 , and the parity is positive. The initial state, with $I=1$ and positive parity, must be singlet and so the only possible initial configurations are 3S_0 and 3D_2 .

At low energies, then, there are three possibilities as far as angular momentum is concerned:

- (a') $2N(^3P_1) \rightarrow D(^3S_1) + \text{meson in } S\text{-state.}$
- (b') $2N(^1D_2) \rightarrow D(^3S_1) + \text{meson in } P\text{-state.}$
- (c') $2N(^1S_0) \rightarrow D(^3S_1) + \text{meson in } P\text{-state.}$

We will denote by δ_0 the ratio of the complex amplitude for process (c') to that for process (b'), the subscript referring to the fact that $J=0$, for (c'). Similarly we will denote by δ_1 the ratio of the amplitude of (a') to that of (b'). These ratios of complex amplitudes are related to the conventional S -matrix elements u_0, u_1, u_2 for $J=0, J=1$, and $J=2$ respectively, by the formulae

$$\delta_0 = - \frac{u_0}{\sqrt{5} u_2}$$

$$\delta_1 = -i \sqrt{3/5} \frac{u_1}{u_2}.$$

Process (a') cannot interfere with the other two in the angular distribution or in the total cross section since (a') involves an initial triplet state and (b') and (c') an initial singlet state. We may therefore discuss S -wave and P -wave production of mesons separately.

The S -wave reaction must be characterized by an isotropic angular distribution and should have a cross section (near threshold) proportional to the meson momentum. At low energies, then, we may write

$$4\pi \frac{d\sigma_{10'}}{d\Omega} (S\text{-wave}) = \alpha\eta \quad 45.$$

where α is independent of angle and energy. A value for the constant α has been determined by Brueckner, Serber, & Watson (3), using experimental data and one theoretical calculation. Let us follow their argument in detail.

Panofsky, Aamodt & Hadley (2) have measured the branching ratio between the two processes $\pi^- + D \rightarrow 2N$ and $\pi^- + D \rightarrow 2N + \gamma$ following the capture of negative pions in deuterium. The pion is presumably absorbed from an S -orbit around the deuteron. Panofsky's ratio is thus about equal to the ratio of the corresponding cross sections for the absorption of slow pions. Using his value of $7/3$ we have

$$\sigma(\pi^- + D \rightarrow 2N) = 7/3\sigma(\pi^- + D \rightarrow 2N + \gamma) \quad 46.$$

for slow pions. If we use (43) and the fact that for slow mesons $p^2 = \mu c^2 M$, where M is the nucleon mass, we have

$$\sigma_{10'} = \sigma(P + P \rightarrow \pi^+ + D) = 3/2\eta^2\mu/M\sigma(\pi^- + D \rightarrow 2N). \quad 47.$$

Now Brueckner, Serber & Watson have estimated by calculation that

$$\sigma(\pi^- + D \rightarrow 2N + \gamma) = 2/3\sigma(\pi^- + P \rightarrow N + \gamma). \quad 48.$$

From detailed balancing we have the further relation

$$\sigma(\pi^- + P \rightarrow N + \gamma) = \frac{2}{\eta^2} \sigma(\gamma + N \rightarrow \pi^- + P). \quad 49.$$

The ratio of π^- -production to π^+ -production in the photopion effect on deuterium is quoted (22) as 1.4, which indicates that

$$\sigma(\gamma + N \rightarrow \pi^- + P) = 1.4\sigma(\gamma + P \rightarrow \pi^+ + N). \quad 50.$$

Combining 46, 47, 48, 49, and 50, we obtain

$$\sigma_{10'} = 14/3(1.4\mu/M)\sigma(\gamma + P \rightarrow \pi^+ + N) \approx \sigma(\gamma + P \rightarrow \pi^+ + N). \quad 51.$$

Using Bernardini's value (16) of η (0.14 millibarn) for $\sigma(\gamma + P \rightarrow \pi^+ + N)$ near threshold and attaching a probable error to cover the various uncertainties in the steps 46 to 50, we come up with something like

$$\sigma_{10'}(S\text{-wave}) = \eta(.14 \pm .05) \text{ millibarn} \quad 52.$$

or

$$\alpha = .14 \pm .05 \text{ millibarn.} \quad 53.$$

Unlike the S -wave, the P -wave contribution to $\sigma_{10'}$ should be characterized by an η^3 -dependence near threshold. The angular distribution is not unique, however, but depends on the complex number δ_0 , the ratio of the amplitude of process c' to that of process b'). It turns out that the angular distribution of the P -wave cross section is of the form $X + \cos^2 \theta$, where X is given by the equation

$$X = \left[\left| \frac{2 - \sqrt{2} \delta_0}{1 + \sqrt{2} \delta_0} \right|^2 - 1 \right]^{-1}. \quad 54.$$

Near threshold, we have then for the dependence of the P -wave cross section on energy and angle the formula

$$4\pi \frac{d\sigma_{10'}}{d\Omega} (P\text{-wave}) = \beta \eta^3 \frac{X + \cos^2 \theta}{X + 1/3}. \quad 55.$$

Taking S - and P -waves together, we obtain for the total cross section

$$\sigma_{10'} = \alpha \eta + \beta \eta^3 \quad 56.$$

and for the angular distribution $A + \cos^2 \theta$, where

$$A = X + \frac{(X + 1/3)}{\eta^2} \frac{\alpha}{\beta}. \quad 57.$$

Now at a finite energy above threshold the parameters α , β , and X may all become functions of η . However, it is theoretically reasonable that as long as the meson wave length is larger than the critical distance R for meson production, *i.e.*, $\eta < 1$, these parameters should vary slowly. We shall try to interpret the experimental data on the assumption that they are constant in the energy range we are considering (Equation 9).

We may then determine β and X by comparing the cross section and angular distribution formulae 56 and 57 with experiment at a single energy. Let us use the data of Crawford & Stevenson (28) (see Table VII), who quote, for $\eta = .58$, the values

$$\sigma_{\text{total}} = .269 \pm .026 \text{ millibarn}$$

and

$$A = .29 \pm .08.$$

If we take $\alpha = .14$ millibarn as in (53) we then find

$$\beta = 1.0 \text{ millibarn} \quad 58.$$

and

$$X = .1. \quad 59.$$

We may now compare our three-parameter semi-empirical formula

$$4\pi \frac{d\sigma}{d\Omega} \left\{ .14\eta + 1.0\eta^3 \frac{(1 + \cos^2 \theta)}{(.1 + 1/3)} \right\} \text{ millibarn} \quad 60.$$

with all the available experimental results in the reactions $P+P\rightarrow\pi^++D$ and $\pi^++D\rightarrow P+P$. Such a comparison is given in Table VII. It should be noted that cross sections for pion absorption have been converted to equivalent cross sections for the inverse process. Similarly the energies have been converted to equivalent proton bombarding energies.

TABLE VII
EXPERIMENTAL DATA ON NUCLEONIC MESON PRODUCTION

Reaction observed	Proton bombarding energy (Mev)	η	Measured value of σ_{10^0} (mb.)	Calculated value of σ_{10^0} (mb.)	Measured value of A	Calculated value of A	Reference
$P+P\rightarrow\pi^++D$	311	.39	$.100\pm.013$.11		.49	(a); (b)
	315	.42	$.133\pm.016$.13		.44	(a); (b)
	321	.46	$.168\pm.018$.16		.38	(a); (b)
	324	.48	$.178\pm.016$.17	$.28\pm.07$.36	(a); (b)
	330	.52	$.228\pm.017$.21		.32	(a); (b)
	332	.54	$.245\pm.013$.23	$.32\pm.05$.31	(b)
	336	.56	$.264\pm.019$.25		.29	(a); (b)
	338	.58	$.269\pm.026$.28	$.29\pm.08$.28	(b)
	340	.59	$.18\pm.06$.29	$.11\pm.06$.27	(c)
$\pi^++D\rightarrow P+P$	341	.59	$.284\pm.050$.29		.27	(d)
	346	.62	$.22\pm.02$.33	$.19\pm.09$.26	(e)
	382	.82	$.66\pm.07$.67	$.26\pm.14$.19	(e)
	413	.96	$.97\pm.10$	1.02	$.18\pm.15$.17	(e)
$P+P\rightarrow\pi^++D$	437	1.05	$1.15\pm.13$	1.30	$.20\pm.02$.15	(f)

References:

(a): Schulz, A. G., *U. S. Atomic Energy Commission Document UCRL-1756* (1952).
(b): Crawford, F. S., and Stevenson, M. L., *U. S. Atomic Energy Commission Document UCRL-2187-2188* (1953).
(c): Cartwright, W. F., Richman, C., Whitehead, M. N., and Wilcox, H. A., *Phys. Rev.*, **91**, 677 (1953).
(d): Clark, D. L., Roberts, A., and Wilson, R., *Phys. Rev.*, **83**, 649 (1951).
(e): Durbin, R., Loar, H., and Steinberger, J., *Phys. Rev.*, **84**, 581 (1951).
(f): Fields, T. H., Fox, J. G., Kane, J. A., Stallwood, R. A., and Sutton, R. B., *Bull. Am. Phys.*, **29** (4) (1954).

The results of Cartwright *et al.* [Table VII, reference (c)] and of Durbin *et al.* [Table VII, reference (e)] at about 340 Mev are the only data in serious disagreement with the semi-empirical formula and are presumably to be thought of as superseded by the results of Crawford & Stevenson.

There are two experimental results on the reaction $N+P\rightarrow\pi^0+D$, which

according to the hypothesis of charge independence, should be absolutely identical with the process $P+P \rightarrow \pi^+ + D$ except for a factor of 1/2 in the absolute cross section (as in Equation 42). Hildebrand (29) gives an angular distribution of $.21 \pm .06 + \cos^2 \theta$ at $\eta = .96$ and Schluter (30) a total cross section of $.6 \pm .2$ millibarn in the range $.85 < \eta < 1.05$. Comparison with equation 60 shows that the charge independence is not violated insofar as the experimental error permits any conclusion.

The determination of the p -wave angular distribution parameter X does not fix the value of the complex number δ_0 , the ratio of $J=0$ and $J=2$ contributions to the production of mesons in p -states. Rather δ_0 is restricted to lying on a circle in the complex plane. In fact,

$$\delta_0 = -\frac{1}{\sqrt{2}}(1 + 3X) + \frac{3}{\sqrt{2}}X(X + 1)e^{i\omega_0} \quad 61.$$

For $X=.1$ we have

$$\delta_0 = -.92 + .70e^{i\omega_0} \quad 62.$$

The value of the phase angle cannot be discovered by means of experiments such as we have discussed. However, it may be found by measuring the polarization of the deuterons in the reaction $P+P \rightarrow \pi^+ + D$ or the asymmetry in the angular distribution of the reaction $\pi^+ + D \rightarrow P+P$ when the target deuterons are polarized. In the case of $P+P \rightarrow \pi^+ + D$, the deuterons associated with the P -wave part of the cross section are polarized perpendicular to the plane of scattering with the degree of polarization P_p given by Watson & Richman (31):

$$P_p = \frac{2\sqrt{X(X+1)} \sin \theta \cos \theta}{X + \cos^2 \theta} \frac{\sin \omega_0}{1 + 2X - 2\sqrt{X(X+1)} \cos \omega_0} \quad 63.$$

where $-1 \leq P_p \leq 1$. The deuterons associated with the S -wave part of the cross section are not polarized perpendicular to the plane of scattering.

We have not yet discussed the relative phase of the amplitudes for S -wave and P -wave meson production. The complex parameter δ_1 gives the ratio of the S -wave amplitude ($J=1$) to the amplitude for P -wave mesons with $J=2$. The absolute magnitude of δ_1 is determined by the relation

$$\frac{|\delta_1|^2}{1 + |\delta_0|^2} = \frac{\alpha}{\beta\eta^2}, \quad 64.$$

but the phase of δ_1 has not yet entered our work. It may be found by measuring the angular distribution of the reaction $P+P \rightarrow \pi^+ + D$ using polarized protons or by measuring the polarization of the protons in the reaction $\pi^+ + D \rightarrow P+P$. (The former experiment is certainly the easier one.) If the incoming proton beam (travelling in the Z -direction) is characterized by a degree of polarization P_1 in the $+X$ -direction then the angular distribution of mesons and deuterons is given by (32).

$$\frac{d\sigma_{10'}}{d\Omega} \propto A + \cos^2 \theta + P_1 Q A \sin \theta \sin \phi \quad 65.$$

where

$$Q = \frac{1}{\sqrt{2}} \frac{2b}{1+b^2} \sin(\psi - \tau_1) \quad 66.$$

if we put

$$\delta_1 = |\delta_1| e^{i\tau_1}, \quad 67.$$

$$b = \frac{|\delta_0 + \sqrt{1/2}|}{|\delta_1|}, \quad 68.$$

and

$$\psi = \arg(\delta_0 + \sqrt{1/2}). \quad 69.$$

So far we have adopted the point of view that quantities do not vary appreciably with energy unless they have to. With such an attitude we would expect that, while $|\delta_1|$ will be inversely proportional to η as in Equation 64, the phase τ_1 should be roughly constant for $\eta \leq 1$. Then the parameter of asymmetry Q will reach its maximum value of $\sqrt{2}/2 \sin(\psi - \tau_1)$ when

$$|\delta_1| = |\delta_0 + \sqrt{1/2}| \quad 70.$$

or, using Equation 64, when

$$\eta = \eta_c \equiv \sqrt{\frac{\alpha}{\beta}} \frac{\sqrt{1 + |\delta_0|^2}}{|\delta_0 + \sqrt{1/2}|}. \quad 71.$$

The energy dependence of the asymmetry becomes, with the use of Equations 64, 66, and 71,

$$Q/Q_{\max} = \frac{2\eta\eta_c}{\eta^2 + \eta_c^2}. \quad 72.$$

The energy of maximum asymmetry corresponds to $\eta_c \approx .77$ no matter what value the phase ω_0 takes. (We put $\alpha/\beta = .14$ as in (60) and use (62) for δ_0 .) The maximum value of $|Q|$ may range from 0 to .71 depending on the phase $\psi - \tau_1$. If this phase happens to be propitious for asymmetry, then the reaction using polarized protons may provide a valuable check on the proportion of S -wave as well as contribute to the determination of the phases.¹⁵

The phases are of considerable importance since they are closely related to the scattering phase shifts of the proton-proton system. We have put

$$\delta_1 = |\delta_1| e^{i\tau_1}. \quad 67.$$

Let us, in the same way, write

$$\delta_0 = |\delta_0| e^{i\tau_0}, \quad 73.$$

Then it can be shown that near the threshold for pion production τ_1 and τ_0

¹⁵ A preliminary result of Marshall, Marshall & de Carvalho (a personal communication) indicates very little asymmetry ($Q = 4 \pm 6$ per cent) around $\eta = 1$; the phase $\psi - \tau_1$ must be very small, and no check on the proportion of S -wave can be made in this way.

can be expressed in terms of the P - P scattering phase shifts $\alpha(^1S_0)$, $\alpha(^1D_2)$, and $\alpha(^3P_1)$ by the relations (where n and n' are integers)

$$\tau_0 = \alpha(^1S_0) - \alpha(^1D_2) + n\pi. \quad 74$$

and

$$\tau_1 = \alpha(^3P_1) - \alpha(^1D_2) + (n' + 1/2)\pi. \quad 75.$$

The arguments leading to Equations 74 and 75 are similar to those used in the appendix to determine the phases of photopion matrix elements in terms of pion-nucleon scattering phase shifts. Equations 74 and 75 begin to fail at energies for which there is appreciable elastic or inelastic scattering of pions by deuterons. If the character of such scattering is understood, then Equations 74 and 75 can be corrected accordingly.

Let us now turn from meson production with deuteron formation to the corresponding reaction in which the two final nucleons are left unbound; in the notation of Equation 42, we pass from $\sigma_{10'}$ to $\sigma_{10''}$. Again the final nucleons have $I=0$ and zero relative orbital angular momentum; they are thus in a 3S_1 state like the deuteron. As in the deuteron reaction, there are three possible processes as far as angular momentum is concerned:

$$\begin{aligned} (a'') \quad & 2N(^3P_1) \rightarrow 2N(^3S_1) + \text{meson in } s\text{-state.} \\ (b'') \quad & 2N(^1D_2) \rightarrow 2N(^3S_1) + \text{meson in } p\text{-state.} \\ (c'') \quad & 2N(^1S_0) \rightarrow 2N(^3S_1) + \text{meson in } p\text{-state.} \end{aligned}$$

We may suppose, in accordance with the point of view we have adopted, that the basic matrix element for each of these processes does not, for a fixed meson momentum, vary very rapidly with the energy of the residual nucleons and may be taken to be roughly constant over the energy range we are considering. In the same spirit, we should say that the matrix element for (a'') should be the same as that for (a') at the same meson momentum, etc. Then we may deduce, from our semi-empirical formula for $\sigma_{10'}$, both the value of $\sigma_{10''}$ and the energy spectrum of the mesons produced in the "unbound" reaction. The only factors we need take into account are the density of final states and the effect of nucleon-nucleon binding in the final states.

Let the bombarding energy be such that a total kinetic energy T_0 is available in the center-of-mass system after the reaction. Of this, an amount $T = \mu c^2(\sqrt{1+\eta^2} - 1)$ is taken by the meson and $E = T_0 - T$ by the internal motion of the two-nucleon system (neglecting its recoil). The differential cross section for the "unbound" reaction should be given by (23).

$$d\sigma_{10''} = (\alpha\eta + \beta\eta^3) \frac{\rho_E dE}{1} \frac{|\psi(R)|^2}{|\psi_D(R)|^2} \quad 76.$$

where $\alpha\eta + \beta\eta^3$ is, of course, the cross section for production of mesons of the same momentum η associated with deuteron formation; $\rho_E dE/1$ is the ratio of the numbers of final states in the "unbound" and "bound" reactions; and $|\psi(R)|^2/|\psi_D(R)|^2$ is the ratio of the squares of the final two-nucleon wave-functions at the critical distance for meson production. This last factor ex-

presses the relative effect in the "unbound" and "bound" reactions of the enhancement of the meson production cross sections by the attractive nucleon-nucleon forces in the final state. If we adopt a simple zero-range model for the nuclear force in the 3S_1 state, with a scattering length a related to the deuteron binding energy B by the equation

$$B = \hbar^2/Ma^2, \quad 77.$$

and if we take $R \approx 0$, then we obtain

$$\left| \frac{\psi(R)}{\psi_D(R)} \right|^2 = \frac{2\pi a \hbar^2}{M(B+E)V} \quad 78.$$

where V is a normalizing volume. The density of states is given, of course, by

$$\rho_E = \frac{V}{(2\pi\hbar)^3} 2\pi M^{3/2} E^{1/2}. \quad 79.$$

We have then (23),

$$\frac{d\sigma_{10}''}{dE} = (\alpha\eta + \beta\eta^3) \frac{1}{2\pi} \left(\frac{E}{B} \right)^{\frac{1}{2}} \frac{1}{E+B} \quad 80.$$

while the cross section for deuteron formation at the same bombarding energy is

$$\sigma_{10}' = \alpha\eta_D + \beta\eta_D^3 \quad 81.$$

with η_D denoting the momentum in units of μc of the meson accompanying the deuteron.

Integration of the meson spectrum (Equation 80) with respect to energy and use of Equation 81 yields a predicted value of the ratio

$$\sigma_{10}''/\sigma_{10} = \frac{\sigma_{10}'''}{\sigma_{10}' + \sigma_{10}''},$$

which is tabulated as a function of nuclear bombarding energy in Table VIII.

In order to find experimental values of the meson spectrum and the "continuum fraction" $\sigma_{10}''/\sigma_{10}$, it is necessary to examine the reactions $P+P \rightarrow \pi^+ + N+P$ or $N+P \rightarrow \pi^0 + N+P$ and to allow for the fact that the cross section for the former is $\sigma_{10} + \sigma_{11}$ and for the latter $\sigma_{10}''/2 + \sigma_{01}/2$ rather than just σ_{10}'' and $\sigma_{10}''/2$ respectively. Fortunately both σ_{11} and σ_{01} are small in our energy region (see below); in Table VIII the experimental ratios need never be corrected by more than 10 per cent on this account. In those cases in which the quantity measured is σ_{10} rather than the "continuum fraction," the fraction is computed using the semi-empirical formula 60 for σ_{10}' . The agreement between measured and predicted ratios is fair, but the predicted values appear to be uniformly low, insofar as the large experimental errors permit any conclusion.

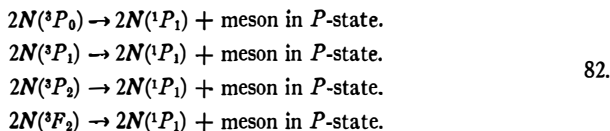
Those experiments which throw light on the continuum meson spectrum predicted in 72 have had, so far, insufficiently good resolution to make comparison with the theory worthwhile. They do not appear, at least, to be in

TABLE VIII
THE "CONTINUUM FRACTION"

Quantity measured	Value of ratio* $\sigma_{10}''/\sigma_{10}$	Pre- dicted ratio*	Bom- barding energy in Mev	Reference
$\sigma_{10}''/\sigma_{10}(P+P)$	35 ± 10	20	341	Cartwright, W. F., (Private communication to A. H. Rosenfeld)
$\sigma_{10}''/\sigma_{10}(P+P)$	45 ± 10	20	341	Peterson, V., Iloff, E., and Sherman, D., <i>Phys. Rev.</i> , 84 , 372 (1951)
$\sigma_{10}(P+P)$	40 ± 30	21	345	Passman, S., Block, M. M., and Havens, W. W., Jr., <i>Phys. Rev.</i> , 88 , 1239 (1952)
$\sigma_{10}(P+P)$	40 ± 30	28	365	Passman, S., Block, M. M., and Havens, W. W., Jr., <i>Phys. Rev.</i> , 88 , 1239 (1952)
$\sigma_{10}(P+P)$	55 ± 30	29	381	Passman, S., Block, M. M., and Havens, W. W., Jr., <i>Phys. Rev.</i> , 88 , 1239 (1952)
$\sigma_{10}''/\sigma_{10}(N+P)$	60 ± 15	32	400	Hildebrand, R. H., and Rosenfeld, A. H., Personal communication.
$\sigma_{10}(P+P)$	73 ± 40	36	440	Rosenfeld, A. H., Personal communication

* Ratio shown in per cent.

obvious contradiction. However, the existing experimental evidence on the continuum reaction does not rule out the possibility that the nucleons are left in a P -state in an appreciable fraction of the meson production events, at least near the high end of the energy range we are considering. The strongest of these processes would presumably be those in which the meson is emitted in a P -state:



For such reactions the cross section per unit energy would be proportional to $\eta^3 E^{3/2}$ rather than the expression 80, provided we ignore the effect of nuclear forces in the final state. (The nature of nuclear forces in the P -state is not well understood, but they are probably relatively weak.) On the same assumption the total excitation function for reactions 82 should be proportional to η_{max}^8 , where η_{max} is the maximum value of η for a given bombarding energy.

So far we have examined only the process $I=1 \rightarrow I=0$. Let us take up next the process $I=1 \rightarrow I=1$, which constitutes the whole of the reaction $P+P \rightarrow P+P+\pi^0$. In the process we have studied up to now the dominant

situation is one in which the meson is emitted in a P -state and the nucleons are left in an S -state. Such a situation is forbidden in the present process, $I=1 \rightarrow I=1$. If the nucleons are left in an S -state, it is a singlet S -state. For the meson to be emitted in a P -state, the total angular momentum must be 1 and the parity of the system even. The only initial states of even parity are 1S_0 , 1D_2 , 1G_4 , etc., none of which has angular momentum 1.

The only allowed situation in which the final nucleons are in an S -state is given by

$$(d) \quad 2N(^3P_0) \rightarrow 2N(^1S_0) + \text{meson in } S\text{-state.}$$

Process (d) is characterized, of course, by an isotropic angular distribution. Near threshold the cross section per unit energy must be of the form

$$\frac{d\sigma_{11}(\text{process } d)}{dE} = \text{const. } \eta \frac{E^{1/2}}{E + B'} \quad 83.$$

as in the s -state part of formula 80, with the binding energy B of the deuteron replaced by the energy B' of the virtual 1S_0 state of two neutrons. Since B' is very small (≈ 60 Kev) we shall drop it. Then the total cross section near threshold is of the form

$$\sigma_{11}(\text{process } d) = \text{const. } \eta_{\max}^2. \quad 84.$$

As in the case of σ_{10}'' , we must not disregard the possibility that the nucleons are left in a P -state. This is particularly true of σ_{11} since the process which is dominant in σ_{10}''' (the production of mesons in P -states with the nucleons left in an S -state) is entirely absent in σ_{11} . If the final nucleons are in a P -state and the meson is emitted in a P -state, we have a list of possible processes analogous to 82:

$$\begin{aligned} 2N(^3P_0) &\rightarrow 2N(^3P_1) + \text{meson in } P\text{-state.} \\ 2N(^3P_1) &\rightarrow 2N(^3P_0) + \text{meson in } P\text{-state.} \\ 2N(^3P_1) &\rightarrow 2N(^3P_1) + \text{meson in } P\text{-state.} \\ 2N(^3P_1) &\rightarrow 2N(^3P_2) + \text{meson in } P\text{-state.} \\ 2N(^3P_2) &\rightarrow 2N(^3P_1) + \text{meson in } P\text{-state.} \\ 2N(^3P_2) &\rightarrow 2N(^3P_2) + \text{meson in } P\text{-state.} \\ 2N(^3F_2) &\rightarrow 2N(^3P_1) + \text{meson in } P\text{-state.} \\ 2N(^3F_2) &\rightarrow 2N(^3P_2) + \text{meson in } P\text{-state.} \\ 2N(^3F_3) &\rightarrow 2N(^3P_2) + \text{meson in } P\text{-state.} \end{aligned} \quad 85.$$

For these processes, as for those in Equation 28, we should expect a cross section per unit energy proportional to $\eta^3 E^{3/2}$ and a total cross section proportional to η_{\max}^8 . Whereas in the case of σ_{10}'' the η_{\max}^8 term in the total cross section has not been detected experimentally with any certainty (it is presumably masked by the dominant reaction), there is strong experimental evidence for this term in σ_{11} . The reaction $P + P \rightarrow \pi^0 + P + P$ has been investigated by Mather & Martinelli (33) at 341 Mev ($\eta_{\max} = .66$) and by Marshall & Marshall (34) at 430 Mev ($\eta = 1.11$). The reported values for the

total cross section are $.010 \pm .003 \text{ mb.}$ and $.45 \pm .15 \text{ mb.}$ respectively. These results are consistent with a pure η_{\max}^8 law of the form

$$\sigma_{11} = (.2\text{mb})\eta_{\max}^8. \quad 86.$$

In 86, we have completely ignored process (d), in which the nucleons are left in an *S*-state and the cross section varies as η_{\max} . That process (d) is probably not absent, we shall see below. It should be important in the total cross section only at low energies, however, where the cross section is very small and measurement difficult.

We come now to the reaction $I=0 \rightarrow I=1$. In this reaction, if the nucleons are to be left in an *S*-state, the only possible processes are

$$(e) \quad 2N(^3S_1) \rightarrow 2N(^1S_0) + \text{meson in } P\text{-state, and}$$

$$(f) \quad 2N(^3D_1) \rightarrow 2N(^1S_0) + \text{meson in } P\text{-state.}$$

For these we should expect, near threshold, a cross section per unit energy of the form

$$\frac{d\sigma_{01}(\text{processes } e \text{ and } f)}{dE} = \text{const. } \eta^8 \frac{E^{1/2}}{E + B'}. \quad 87.$$

As in 83, we may drop B' ; we obtain in this case a total cross section obeying the law

$$\sigma_{01}(\text{processes } e \text{ and } f) = \gamma\eta_{\max}^4. \quad 88.$$

If the final nucleons are in a *P*-state and the meson emitted in a *P*-state, the possibilities are:

$$2N(^1P_1) \rightarrow 2N(^3P_0) + \text{meson in } P\text{-state.}$$

$$2N(^1P_1) \rightarrow 2N(^3P_1) + \text{meson in } P\text{-state.}$$

$$2N(^1P_1) \rightarrow 2N(^3P_2) + \text{meson in } P\text{-state.}$$

$$2N(^1F_3) \rightarrow 2N(^3P_2) + \text{meson in } P\text{-state.}$$

89.

For these, we should expect, as usual, a total cross section varying like η_{\max}^8 .

Unfortunately, experimental evidence on σ_{01} is available only at a single energy. It can be seen from Equation 42 that σ_{01} is always observed experimentally in conjunction with either σ_{10}'' or σ_{11} . In fact,

$$\sigma(N + P \rightarrow \pi^+ + N + N) = \sigma(N + P \rightarrow \pi^- + P + P) = 1/2\sigma_{11} + 1/2\sigma_{01}. \quad 90.$$

and

$$\sigma(N + P \rightarrow \pi^0 + N + P) = 1/2\sigma_{10}'' + 1/2\sigma_{01}. \quad 91.$$

A total cross section for $N + P \rightarrow \pi^+ + N + N$ or $N + P \rightarrow \pi^- + P + P$ at around 405 Mev ($\eta = .915$) is reported by Yodh (35) who finds $.22 \pm .07 \text{ mb.}$ (See also 36.) If we now estimate σ_{11} at this energy by means of an interpolation formula like 86, and then use 90 to find σ_{01} , we obtain a very rough value of $.3 \text{ mb.}$ for σ_{01} at $\eta = .915$. Despite the enormous probable error to be attached to this value of σ_{01} , its smallness in comparison to the value of σ_{10}'' at the same energy ($\geq 1 \text{ mb.}$) is certainly significant.

Since we have no information concerning σ_{01} at other energies, we cannot check whether processes e and f effectively dominate the processes in 89. Let us tentatively assume, however, that the final nucleons are left primarily in S -states in which case we have for the constant in Equation 88

$$\gamma \approx 0.5mb. \quad 92.$$

It should be noted that the relations 90 and 91 apply to total cross sections. In the case of 91, no interference is possible between the two isotopic spin processes under the hypothesis of charge independence, and so a similar formula obtains for the differential cross sections. Moreover, there is no forward-backward asymmetry in the angular distribution. (In a given isotopic spin state, the neutron and proton behave like indistinguishable particles.) In the reaction described by 90, however, interference between the two processes is possible and can lead to forward-backward asymmetry.

The angular distribution of the reaction $N+P \rightarrow \pi^- + P+P$ has been investigated by Yodh (35) and by Wright & Schluter (36), and forward-backward asymmetry has been discovered. If the reaction $I=0 \rightarrow I=1$ does leave the final nucleons mainly in S -states, then the reaction $I=1 \rightarrow I=1$ must also do so at least part of the time in order to produce such interference. Thus the process (d) above probably occurs with appreciable strength. It is difficult, however, on the basis of existing experimental evidence, to say anything quantitative.

Let us now review the principal features of the experimental data, interpreted in the manner we have described. It appears that low energy meson production phenomena have the following properties:

Nucleons left in S -states:

$I = 1 \rightarrow I = 0$: Meson production in P -states large, in S -states small. P -wave angular distribution is $\approx .1 + \cos^2 \theta$.

$I = 0 \rightarrow I = 1$: Meson production in P -states, small, in S -states forbidden.

$I = 1 \rightarrow I = 1$: Meson production in P -states forbidden, in S -states small.

Nucleons left in P -states:

Need be invoked only in case $I=1 \rightarrow I=1$. Not well understood.

In particular there are at present four important experimentally determined quantities that must be predicted by a theory that is to transcend the simple theoretical picture we have used:

(A) The ratio of S -wave to P -wave meson production in the reaction $I=1 \rightarrow I=0$.

$$\alpha/\beta \approx 1/7. \text{ (See Equations 45, 53, 55, and 58.)}$$

(B) The P -wave angular distribution parameter in the reaction $I=1 \rightarrow I=0$.

$$X \approx .1. \text{ (See Equations 55 and 59.)}$$

(C) The ratio of P -wave meson production in the reaction $I=0 \rightarrow I=1$ to that in the reaction $I=1 \rightarrow I=0$.

$$\sigma_{01}/\sigma_{10}'' \lesssim .3 \text{ (See discussion preceding Equation 92.)}$$

(D) The absolute cross section for meson production.

$$\beta \simeq 1mb. \text{ (See Equations 55 and 58.)}$$

If we try to predict these quantities on the basis of some version of pseudoscalar meson theory, we see that the fourth one is related directly to the coupling constant and thus to the details of the theory (37). The other three also involve to some extent the details of the theory, but we can nevertheless obtain some understanding of their magnitudes in a fairly simple manner.

With regard to (A), it has been observed in the introduction that pseudoscalar mesons should interact with nucleons strongly in P -states, while the S -state interaction is in the nature of a recoil correction. Near the threshold for meson production, the basic matrix element for the emission of a pseudoscalar meson in a P -state is proportional to $\delta \cdot \mathbf{k}$, where δ is the spin of the emitting nucleon and $\mathbf{k} = \mu \mathbf{v}$ is the meson momentum. If we now insert a correction for the motion of the nucleon, we must replace \mathbf{v} by $\mathbf{v} - \langle \mathbf{v} \rangle_n$, where $\langle \mathbf{v} \rangle_n$ is the average of the initial and final velocities of the emitting nucleon. (The principle of invariance under Galilean coordinate transformations requires that matrix elements depend on relative and not absolute velocities.) Now the final nucleon velocity is close to zero, while the initial one \mathbf{v}_0 (in the center-of-mass system) satisfies

$$Mv_0^2 \simeq \mu c^2 \quad 93.$$

since the kinetic energy of the colliding nucleons is transformed into meson rest energy. Thus recoil corrections add to the term $\delta \cdot \mathbf{k}$ (representing P -wave meson production) a term $-\delta(\mu v_0/2)$ representing S -wave meson production and the ratio of intensities is of the order of $\mu^2 v_0^2 / 4k^2 = (\mu/4M)(1/\eta^2)$, which should correspond roughly to $\alpha/\beta\eta^2$. We see, then, the α/β should be of the order of μ/M . This conclusion is borne out by detailed meson theories.

With regard to (B) and (C), an explanation of the smallness of $\sigma_{01}/\sigma_{10}''$ and the closeness of X to $1/3$ (it could have, in principle, any value between 0 and ∞ !) has been offered by Aitken *et al.* (37). They make use of the strong pion-nucleon attraction in the $(3/2, 3/2)$ state, which has been discussed in earlier sections. The basic idea is that if in the final state of a meson production process the pion and one of the nucleons can form a $(3/2, 3/2)$ state the strong attractive forces enhance the matrix element for the process involved. (In a similar way, we have seen that the nucleon-nucleon forces serve to enhance those reactions in which the final nucleons are left in an S -state.)

Now there are four processes which contribute to P -wave meson production (with the nucleons left in a continuum S -state); we have labeled them as b'' , c'' , e and f . Of these, b'' and c'' contribute to σ_{10}'' , while e and f contribute to σ_{01} . Processes e and f cannot be enhanced by the $(3/2, 3/2)$ effect since the total isotopic spin of the system in these cases is 0, while a nucleon and a pion in an $I=3/2$ state plus another nucleon can have $I=1$ or 2 only.

Process c'' is not enhanced either, provided that the two nucleons are close together when the meson is produced, compared to the wave-length of the meson. (We have used this assumption before in assigning low angular momentum values to the outgoing particles.) The $J=0$ state then cannot result in the formation of a pion and nucleon in a $J=3/2$ state plus another nucleon in an S -state. The same argument applies to process c' , which results in deuteron formation.

In the case of process b'' and b' , enhancement occurs, and so we can understand that $\sigma_{01} \ll \sigma_{10}''$ and that the angular distribution of the $I=1 \rightarrow I=0$ process is close to the form characteristic of b'' or b' alone, i.e., $1/3 + \cos^2\theta$. Note that if we accept this explanation of the angular distribution, the phase angle ω_0 in (62) must be close to 0° . Aitken *et al.* (37) have formed this argument more quantitatively, using detailed meson theory.

MESON THEORY¹⁶

Meson theory, since the original proposal of Yukawa, has been constructed by analogy with quantum electrodynamics. Just as the primary process in electrodynamics is the virtual emission or absorption of a single photon by an electron, so in meson theory it is the virtual emission or absorption of a single pion by a nucleon. The "bare" nucleon (uncoupled to the pion field) is supposed to be described, like the electron, by Dirac's equation. The pseudoscalar pion field operator $\phi(\mathbf{x}, t)$ is analogous to the vector (spin 1) field operators $A_\mu(\mathbf{x}, t)$ representing the quantized potentials of the electromagnetic field. As $A_\mu(\mathbf{x}, t)$ is coupled to the vector Dirac operator γ_μ for the electron, so $\phi(\mathbf{x}, t)$ may be coupled to the pseudoscalar Dirac operator γ_5 for the nucleon (PS or pseudoscalar coupling) or else the gradient of ϕ , $\partial\phi/\partial x_\mu(\mathbf{x}, t)$, may be coupled to the pseudovector Dirac operator $\gamma_5\gamma_\mu$ for the nucleon (PV or pseudovector coupling). No simple coupling of the pion field other than these two has been suggested. (It should be noted that to describe the three charge states of the pion, a three-component field ϕ_i is required, the three components forming a vector in the space of isotopic spin. In order to fulfill the requirements of charge independence, a "symmetrical" theory is used (5) in which the components ϕ_i are coupled to the components τ_i of the nucleon isotopic spin.)

For each of the two couplings a fully relativistic theory of pion-nucleon interactions can be set up and studied by the perturbation method, i.e., expansion of observable quantities in powers of the coupling constant $g^2/4\pi\hbar c$ (for PS) or $f^2/4\pi\hbar c$ (for PV) for the interaction. As in quantum electrodynamics, the coefficients in such an expansion turn out to be infinite (after the lowest power of the coupling constant). In electrodynamics these infinities disappear when the results are re-expressed in terms of the observed mass and charge of the electron (the so-called mass and charge renormalization). The

¹⁶ In this section we shall usually put $\hbar=c=1$.

same is found to be true in the PS theory of pions (38).¹⁷ In the PV theory, however, infinities remain and the relativistic theory must be modified if finite results are to be obtained. A suitable modification is the introduction of a finite radius a for the bare nucleon, of the order of the nucleon Compton wavelength \hbar/Mc . All integrals over virtual pion momenta are then cut off at a momentum of \hbar/a or an energy of $\omega_{\max} = \sqrt{\mu^2 c^2 + \hbar^2/a^2}$.

When the cut-off PV theory is employed for calculations, it is usual to introduce the static approximation, that is, to treat nucleons as fixed and ignore all or most effects of nucleon recoil. This approximation is not absolutely necessary, since in principle a relativistically invariant cut-off can be used and the effects of nucleon motion in the PV theory retained. However, it is usually felt that, since a cut-off must be used, recoil is probably badly treated anyway and might just as well be left out.

The static, cut-off PV theory has been treated in weak and strong coupling by several authors [for references to earlier work see (39); for recent work see (40)]. It has been found that while very weak and very strong coupling are both incompatible with experimental data on pion phenomena, the cases of moderately weak and moderately strong coupling both present features which are strongly suggestive of the experimental situation, particularly the presence of strong attractive forces between pions and nucleons in the $(3/2, 3/2)$ state. The case of moderately weak coupling has recently been studied in great detail by Chew (40) and the results of calculation compared with experiment (41).¹⁸ He finds that with a coupling constant $f^2/4\pi\hbar c = .058$ and a cut-off energy $\omega_{\max} = .84 mc^2$ it is possible to obtain rough quantitative agreement with experimental data on pion scattering and the photopion effect at meson energies < 250 Mev and the anomalous magnetic moments of neutron and proton, all phenomena involving a single nucleon. We shall refer to his approach as the Chew theory; let us examine it in some detail.

The Hamiltonian of the Chew theory is of the form

$$H = H_{\pi} + \frac{f}{\mu} \sum_i \tau_i \cdot \int \rho(\mathbf{x}) \nabla \phi_i(\mathbf{x}) d^3x + M. \quad 94.$$

Here H_{π} is the field Hamiltonian of the pion and $\rho(\mathbf{x})$ is the function describing the nucleon as a source with finite extension; ρ is roughly characterized by the cut-off energy ω_c and the condition

$$\int \rho(\mathbf{x}) d^3x = 1. \quad 95.$$

The nucleon is placed at the origin.

This Hamiltonian need not be thought of as an approximation to that of the relativistic PV theory. If we wish to construct a charge-symmetric

¹⁷ In the case of PS meson theory, it is in fact necessary to introduce and renormalize one more parameter, describing the scattering of mesons by mesons. (See 47.)

¹⁸ We wish to thank Professor Chew for sending us many papers in advance of publication.

theory of the interaction of pseudoscalar mesons with a stationary nucleon, and if the interaction is to be linear in the meson field so that only a single meson is created or destroyed in an elementary act, we are led almost uniquely to 94.

If interaction with the photon field is to be introduced, then additional terms must be included in the Hamiltonian:

$$H_{\text{add.}} = H_{ph} - \int j \cdot A d^3x + \frac{fe}{\mu} \int d^3x \rho(x) \delta \cdot A(\phi_1 \tau_2 - \phi_2 \tau_1). \quad 96.$$

Here the first term is the field Hamiltonian of the photon field; the second is the interaction of the meson current with the electromagnetic potential; the third is a term describing direct photomeson production; it arises from the requirement of gauge invariance. We have omitted all Coulomb interactions, the interaction of the Dirac magnetic moment of the proton with the photon field, and terms required by gauge-invariance inside the nucleon.

It is convenient to determine the coupling constant from experimental data on the photo-pion effect near threshold. In the Chew theory, this effect arises at low energies entirely from the third term in 96; moreover, it charge renormalization is performed in the conventional way, then the formula obtained by lowest-order perturbation theory is exact:

$$\sigma(\gamma^\pm) = 8\pi \left(\frac{e^2}{4\pi\hbar c} \right) \left(\frac{f^2}{4\pi\hbar c} \right) \left(\frac{\hbar}{\mu c} \right)^2 \frac{\eta}{\nu} \quad (\eta \ll 1). \quad 97.$$

Here η and ν are, as in the section on PHOTOPRODUCTION OF π -MESONS FROM NUCLEONS, the momenta of meson and photon respectively in units of μc . Comparison with the experimental results of Bernardini & Goldwasser (16) gives for the coupling constant the value $f^2/4\pi\hbar c = .038$. Chew points out, however, that it is possible to correct formula 97 for certain kinematic effects of nucleon motion that are not included in the Chew theory itself. The resulting formula is

$$\sigma(\gamma^\pm) = 8\pi \left(\frac{e^2}{4\pi\hbar c} \right) \left(\frac{f^2}{4\pi\hbar c} \right) \left(\frac{\hbar}{\mu c} \right)^2 \frac{\eta}{\nu} \left(1 \mp \frac{\omega}{2Mc^2} \right)^2 \left(1 + \frac{\nu\mu}{M} \right)^{-1} \left(1 + \frac{\omega}{Mc^2} \right)^{-1} \quad (\eta \ll 1). \quad 98.$$

where ω is the pion energy $\mu c^2 \sqrt{1+\eta^2}$. If formula 98 rather than 97 is compared with the data of Bernardini & Goldwasser the coupling constant turns out to be $.058 \pm .015$. Equation 98 predicts for $\sigma(\gamma^-)/\sigma(\gamma^+)$ near threshold a value 1.3 in good agreement with the experimental value quoted in Equation 20.

Using the value we have just found for the coupling constant, Chew (40, 41) and others have calculated the p -wave phase shifts for pion-nucleon scattering. (It should be noticed that the Chew theory predicts no scattering in any states other than P -states. This is a rather serious difficulty, especially since it seems impossible to interpret the observed S -scattering as a recoil

effect; we have, in fact, at present no theoretical understanding of the S -wave in pion-nucleon scattering.)

In the P -wave phase shift calculations, it is found that perturbation theory is inadequate, especially for the $(3/2, 3/2)$ state. Although the coupling is moderately weak and second and fourth order perturbation theory suffice more or less for calculating the "effective potential" in pion-nucleon scattering, this "potential" is not in itself weak enough in all states to be treated in Born approximation. The word "potential" is placed in quotation marks because the actual operator calculated is not a static potential $V(r)$ but something rather highly momentum- and energy-dependent. It was Tamm (42) and Dancoff (43) who first suggested that in problems such as these it would be better to calculate the nonstatic "potential" by perturba-

TABLE IX
 P -WAVE PION-NUCLEON PHASE SHIFTS AS CALCULATED BY CHEW

E_{lab} (Mev)	η	α_{33}	$\alpha_{31} = \alpha_{13}$	α_{11}
38	.67	3.7°		
57	.83	8.0°	-1.0°	-2.3°
78	.98	14.7°		
99	1.12	24.9°	-2.0°	-4.3°
122	1.25	39.4°		
144	1.38	56.9°	-3.3°	-6.4°
167	1.50	73.2°		
190	1.62	85.8°	-4.8°	-8.4°
215	1.73	94.6°		
240	1.85	99.5°	-6.3°	-10.3°

tion theory and then calculate the scattering phases exactly than to expand the phase shifts directly in powers of the coupling constant. The method is now referred to as the Tamm-Dancoff or T-D method. It is not applicable when the coupling strength is so large that even the "potential" cannot be treated correctly by perturbation theory.

Using a slight modification of the T-D method and solving for the phase shifts by numerical methods, Chew (40), Gammel (44), and Salzman & Snyder (45) have found, with $\omega_{\max} \cong .84 mc^2$, values of the phase shifts that are in satisfactory agreement with the present experimental evidence. Chew's phase shifts are listed in Table IX. It is seen that the scattering in the $(3/2, 3/2)$ state is attractive and "resonant"; in the other states it is repulsive and quite weak. It would presumably be unreasonable to compare the static cut-off theory with experiment at energies much greater than those listed, since recoil effects and the detailed nature of the cut-off should begin to enter the picture.

We may now return to the photopion effect and inquire what the Chew theory predicts for energies at which formula 98 no longer applies. Unfortu-

nately the relevant calculations are not yet in fully satisfactory shape; moreover, a static theory is necessarily ambiguous with regard to the treatment of the Dirac magnetic moment of the proton. Chew (41) reports, however, that preliminary calculations seem to agree fairly well with experimental data in the "resonance" region. Certainly the fact that the $(3/2, 3/2)$ resonance occurs in the theory will cause to be applicable the kind of analysis we have used in the section on **PHOTOPRODUCTION OF π -MESONS FROM NUCLEONS**.

The meson-current contribution to the proton and neutron magnetic moments (necessarily equal and opposite in the two cases) has been calculated in perturbation approximation by Chew and collaborators (41). The result, with the same parameters as above, is ± 1.15 nuclear Bohr magnetons in second order and ± 1.48 when fourth order corrections are added. The experimental values of the anomalous moments are, of course, $+1.79$ and -1.91 for proton and neutron respectively.

In addition to Chew's program of investigation of one-nucleon problems, the static PV theory has been applied to the two-nucleon problem, *i.e.*, the meson theory of nuclear forces. The second- and fourth-order static potentials between a pair of nucleons have been calculated by Taketani *et al.* (46), Feynman & Lopes (47), Brueckner & Watson (48), and others. These potentials are highly singular at the origin and a boundary condition at small distances must be introduced if the Schrödinger equation is to be solved. The usual choice is the vanishing of the two-nucleon wave function at a separation of around $1/2\hbar/\mu c$ [the so-called hard core, discussed originally by Jastrow (49)]. With such an assumption and a coupling constant not substantially different from Chew's, it has been found that all the experimental parameters relating to the low energy two-body problem can be predicted with fair accuracy. Moreover it has lately been shown by Taketani *et al.* (50) and by Brueckner (51) that the qualitative features of nucleon-nucleon scattering up to 90 Mev can be well understood in terms of the same potential. It is important to show, of course, that higher order effects do not spoil the agreement with experiment. Present indications (48, 50) are that sixth and higher order potentials, while very strong, are also of very short range, and may not be of great importance outside the "core." The same may be true of effects due to the "new particles."

Let us now turn from the static cut-off PV theory to the fully relativistic PS theory, which gives finite results without a cut-off. The behavior of the PS theory is not so well understood as that of the other theory we have been discussing. It is quite certain that if the coupling constant is so small that straight perturbation theory is applicable, then the PS theory disagrees with observation. Also, no strong coupling approximation has ever been found that remotely resembles the experimental situation. Various approximations have been suggested for which validity is claimed in the range of intermediate coupling strength. It is at present doubtful, however, whether any of these

really gives an accurate picture of the predictions of the PS theory. As to the agreement of these approximations with experimental data, it is probably fair to say that they agree with experiment just insofar as they resemble the static cut-off theory of Chew. It may indeed be true that the Chew theory is a fair approximation to the PS theory with intermediate coupling strength, but a demonstration of that proposition has yet to be given.

The coupling constant that must be used in the PS theory if it is to agree with experiment can be determined by a method very similar to that employed above for the Chew theory, i.e., comparison with the photopion effect near threshold. Lowest-order perturbation theory gives for the PS theory Equation 98 with $f^2/4\pi\hbar c$ replaced by $(\mu/2M)^2 g^2/4\pi\hbar c$, where $g^2/4\pi\hbar c$ is the PS coupling constant. Now in PS theory this formula is not correct to all orders as Equation 97 is in the Chew theory. However, Kroll & Ruderman (52) have shown that it is correct to all orders except for terms of the order of μ/M ; they present arguments, too, to show that such terms are indeed small.¹⁹ If we determine $g^2/4\pi\hbar c$ in this way, we find a value of about 10. It is clear why perturbation theory is not altogether satisfactory for calculating other processes!

In order to gain some insight into the structure of the PS theory, we may apply it to the method of Foldy & Wouthuysen (53). They eliminate from the Hamiltonian by a succession of canonical transformations the odd Dirac matrices (those anticommuting with β) in successive approximations in $1/M$. The original PS Hamiltonian, which we write for simplicity in the "one-particle" form, is

$$H_{PS} = H_\pi + \boldsymbol{\alpha} \cdot \boldsymbol{p} + \beta M + ig\beta\gamma_5 \sum_i \tau_i \phi_i(\boldsymbol{x}), \quad 99.$$

where \boldsymbol{x} and \boldsymbol{p} are the coordinate and momentum respectively of the nucleon. After transformation, to first order in $1/M$, we have

$$H_{PS}' = H_\pi + \frac{g}{2M} \boldsymbol{\sigma} \cdot \boldsymbol{\Delta} \sum_i \tau_i \phi_i(\boldsymbol{x}) + M + \frac{g^2}{2M} \sum_i \phi_i^2(\boldsymbol{x}) + \frac{\boldsymbol{p}^2}{2M}. \quad 100.$$

If we compare 100 with Equation 94 for the Chew theory, putting $f = g\mu/2M$, we see that the first three terms in 100 correspond exactly to the Chew Hamiltonian, although the cut-off is not present. However, the recoil kinetic energy term $\boldsymbol{p}^2/2M$ provides a cut-off of much the same kind after renormalization. The chief difference lies in the term

$$\frac{g^2}{2M} \sum_i \phi_i^2(\boldsymbol{x}),$$

which corresponds to the interaction of S -wave mesons with the nucleon, through scattering and through creation and destruction of pairs of mesons.

¹⁹ Note that the correctness of the perturbation theory result as $(\mu/M) \rightarrow 0$ depends on the conventional choice of charge renormalization procedure.

Further, the recoil term couples the S -wave and P -wave mesons to some extent. At first sight, it would seem that Equation 100 must be an improvement over 94, since it gives similar results for P -wave mesons but predicts in addition some S -wave scattering, the lack of which was a flaw in the Chew theory. If we calculate the S -wave scattering using formula 100, though, we find it in disagreement with experiment. First of all, perturbation theory gives a very strong S -wave scattering which is totally at variance with experience. A refined treatment by Wentzel (54) shows that this effect is very strongly damped by higher order processes; the magnitude of the S -wave scattering is then similar to that observed. However, the isotopic spin and energy dependence of the S -scattering are still in contradiction with experiment. The success of 100 is not significantly different from that of the Chew theory.

However, 100 represents only an approximation to the PS theory. One may hope, therefore, that a correct treatment of the full PS Hamiltonian will give at least as good agreement with the known properties of pions in P -states and, in addition, explain such effects as the weak, isotopic spin-dependent S -wave scattering. A research project has been undertaken at Cornell by Bethe, Dyson and others (55) to determine whether this is in fact the case. They make use of the Tamm-Dancoff method, which is unfortunately not of proved validity in the case of PS theory with a coupling constant of 10. Calculations are in progress of cross sections for meson scattering and photomeson production and preliminary reports of the work seem encouraging as regards agreement with experiments on P -wave pions.

If the full PS theory is to prove correct, one other feature of the approximate Hamiltonian 100 must be preserved; that is the damping of S -wave scattering. Some indication that this may occur in the relativistic PS theory has been provided by a very rough calculation of Brueckner, Gell-Mann & Goldberger (56).

Bethe (55) has expressed confidence that the PS theory will turn out to give a correct description of pion phenomena at moderately low energies. He attaches great significance to the fact that the PS theory is the only known relativistically invariant theory of pseudoscalar mesons that gives finite results after renormalization without a cut-off. Chew (41) is inclined, on the other hand, to believe that attempts to refine the static cut-off PV theory must be more ambitious than the use of the PS theory, and must ultimately involve a description of the "new unstable particles." It is indeed difficult to believe that such a description is possible within the framework of any existing theory.

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APPENDIX:

CALCULATION OF THE COMPLEX PHASES OF THE MATRIX
ELEMENTS FOR PHOTOPRODUCTION

We indicate briefly the method employed in references of footnote 11 for determining the phases in Equations 19 and 20. Consider the transition between eigenstates of a Hamiltonian H_0 induced by an interaction V . Then the integral equation satisfied by the reaction matrix K is*

$$K = V + V \frac{1}{E - H_0} K, \quad \text{A-1.}$$

where E is the energy of the system. If V and H_0 are invariant under the Wigner time reversal operator K , then so is K ; i.e.

$$KKK^{-1} = K. \quad \text{A-2.}$$

The integral equation relating K to the scattering amplitude T is*

$$T = K + iKT, \quad \text{A-3.}$$

where all quantities are here restricted to the energy shell (in contrast to those of Equation A-1). Let us denote the eigenfunctions of H_0 by

$$\phi_{rj}^m$$

where r denotes the channel parameter (i.e., the types of states into which the scattering may lead) and j is the total angular momentum with z -component m . (There will in general be other eigenvariables which we shall ignore for the sake of brevity.) We may expand K in terms of the ϕ 's:

$$K = \sum_{r,r'} \sum_m (r/K/r') \phi_{rj}^m \phi_{r'j}^{*m}. \quad \text{A-4.}$$

Here $(r/K/r')$ is a function only of energy and r and r' . If we choose the most common representation, then

$$K\phi_{rj}^m = (i)^{2m} \phi_{rj}^{-m} \quad \text{A-5.}$$

and (since K involves complex conjugation)

$$KKK^{-1} = \sum_{r,r'} \sum_m E(r/K/r')^* (i)^{2m} (-i)^{2m} \phi_{rj}^{-m} \phi_{r'j}^{*m}. \quad \text{A-6.}$$

Making use of Equation A-2 we see that

$$(r/K/r')^* = (r/K/r') \equiv K_0. \quad \text{A-7.}$$

Thus the submatrix $(r/K/r')$ is real and symmetric (since it is hermitean), a result which depends upon choosing the ϕ 's to satisfy equation A-5.

* See, for instance, M. Gell-Mann & M. L. Goldberger, *Phys. Rev.*, **91**, 398 (1953) for the general theory of scattering processes.

To illustrate the implications of equation A-7 let us consider meson-nucleon scattering for a pure (j, l) state and photoproduction to the same state of the meson and nucleon. Then

$$K_0 = \begin{pmatrix} 0 & \gamma \\ \gamma & \tan \delta \end{pmatrix} \begin{matrix} (\gamma\text{-ray-nucleon state}) \\ (\text{pion-nucleon state}), \end{matrix} \quad \text{A-8.}$$

where δ is the scattering phase shift and γ is the strength of the coupling to the γ -ray channel. We consider γ to be small (since it involves an electromagnetic interaction) and neglect the γ - γ scattering by a nucleon. (This is the reason that the element in the upper left hand corner of K_0 is zero.)

Returning to Equation A-3, we expand T in the form of Equation A-4, letting

$$T_0 \equiv (r/T/r').$$

In terms of T_0 and K_0 , Equation A-3 is

$$T_0 \equiv K_0 + iK_0T_0,$$

or

$$T_0 = (1 - iK_0)^{-1}K_0. \quad \text{A-9.}$$

The matrix products are easily evaluated and we find

$$T_0 = \begin{pmatrix} 0 & e^{i\delta}\gamma \cos \delta \\ e^{i\delta}\gamma \cos \delta & e^{i\delta} \sin \delta \end{pmatrix}. \quad \text{A-10.}$$

Here the off diagonal matrix elements represent a single multipole matrix element for photoproduction. Since δ and γ are real, this has the form

$$e^{i\delta} \text{ times a real quantity.}$$

This result has a number of implications for photoproduction. These have been derived in detail (see footnote 11, page 241). We quote them here. The multipole expressions of Equations 26 are expressed in terms of the amplitude for transitions to pure I -states. Then (here the α 's are the pion-nucleon scattering phase shifts evaluated at the energy of pion and nucleon in the final state)

$$\begin{aligned} E_1^+ &= e^{i\alpha_{31}}\sqrt{2} E_1^{(3)} + e^{i\alpha_1} \frac{1}{\sqrt{2}} [E_1^{(1)} - 2\delta E_1^{(1)}] \\ E_1^0 &= e^{i\alpha_{31}}2E_1^{(3)} - e^{i\alpha_{11}}1/2[E_1^{(1)} - 2\delta E_1^{(1)}] \\ M_1(1/2)^+ &= e^{i\alpha_{31}}\sqrt{2} M_1(1/2)^{(3)} + e^{i\alpha_{11}} \frac{1}{\sqrt{2}} [M_1(1/2)^{(1)} - 2\delta M_1(1/2)^{(1)}] \\ M_1(1/2)^0 &= e^{i\alpha_{31}}2M_1(1/2)^{(3)} - e^{i\alpha_{11}}1/2[M_1(1/2)^{(1)} - 2\delta M_1(1/2)^{(1)}] \\ M_1(3/2)^+ &= e^{i\alpha_{33}}\sqrt{2} M_1(3/2)^{(3)} + e^{i\alpha_{13}} \frac{1}{\sqrt{2}} [M_1(3/2)^{(1)} - 2\delta M_1(3/2)^{(1)}] \\ M_1(3/2)^0 &= e^{i\alpha_{33}}2M_1(3/2)^{(3)} - e^{i\alpha_{13}}1/2[M_1(3/2)^{(1)} - 2\delta M_1(3/2)^{(1)}] \\ E_2^+ &= e^{i\alpha_{33}}\sqrt{2} E_2^{(3)} + e^{i\alpha_{13}} \frac{1}{\sqrt{2}} [E_2^{(1)} - 2\delta E_2^{(1)}] \\ E_2^0 &= e^{i\alpha_{33}}2E_2^{(3)} - e^{i\alpha_{13}}1/2[E_2^{(1)} - 2\delta E_2^{(1)}]. \end{aligned} \quad \text{A-11,}$$

The 12 quantities $E_1^{(3)}$, $M_1(1/2)^1$, $\delta E_2^{(1)}$, etc. represent transition amplitudes to pure I -states of the final pion-nucleon system. The superscript is just twice the value of this I -spin. The primary significance of these equations is that the 12 quantities $E_1^{(3)}$, etc., are real functions of the γ -ray energy only.

We obtain the multipole amplitudes for P_γ^- from those for P_γ^+ and for $P_{(\gamma n_0)}$ from those for P_γ^- in Equations A-11 by simply changing the sign of $\delta E_1^{(1)}$, $\delta M_1(1/2)^{(1)}$, $\delta M_1(3/2)^{(1)}$ and $\delta E_2^{(1)}$. Thus the 16 complex multipole amplitudes are expressed in terms of 12 real quantities. For further details see footnote 11, p. 241.

Only $E_2^{(3)}$ and $M_1(3/2)^{(3)}$ contribute to the "resonant state." For $E_\gamma \cong 320$ Mev, these are expected to have the form (23)

$$M_1(3/2)^{(3)} \text{ and } E_2^{(3)} = \frac{\sin \alpha_{33}}{\eta^2} \text{ times a real constant,}$$

as written in Equation 33.

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